Building a Knowledge Base and Intellectual Capacity in Mathematics Education: Promises and Challenges

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Outline

• Three Reflections
• New NCTM Research Handbook
• LieCal Project
• Mathematical Model of Marriage
Reflection 1

Math Education Researcher

First, Cross-national Comparative Studies
Second, Curriculum Studies
Third, Mathematical Exploration
Reflection 2
Teacher Educator

• Many Reform Ideas
• Many Theories
MAKE JUST ONE CHANGE

Teach Students to Ask Their Own Questions

DAN ROTHSTEIN and LUZ SANTANA

Foreword by WENDY D. PURIEFOY

“This book begins with the seemingly simple request to get students to ask their own questions, but at heart it's a book about creating a classroom alive with dialogue, inquiry, and respect for students' minds.”

—MIKE ROSE, author of Why School? Reclaiming Education for All of Us.
Practices
5 in Sustainable Outdoor Education
Teaching and Learning

Educational Setting

ZPD

Natural Setting
Reflection 3

NSF Program Director

• Elevator talk

• IES and NSF Common Guidelines for Education Research and Development
Outline

• Three Reflections

• New NCTM Research Handbook

• LieCal Project

• Mathematical Model of Marriage
Research Handbooks

• Cited Very Frequently
• Similar Structure
• Similar Topics
Research Handbooks

• New Topics
• New Sections
• New Consideration of Author Teams
Outline

• Three Reflections
• New NCTM Research Handbook
• LieCal Project
• Mathematical Model of Marriage
Longitudinal Investigation of the Effect of Curriculum on Algebra Learning (LieCal Project)
Project Team

PIs
• Jinfa Cai  John C. Moyer  Ning Wang

Project Coordinators at the Research Site
• Pat Bolter/Victoria Robison

Research Associates/Specialists/Graduate Assistants
• Bikai Nie  Tammy Garber
• Tony Freedman  Stephen Hwang
• Yuichi Handa  Connie Laughlin
• Patrick Hopfensperger  Yue Zeng  Steve Silber
• Teresa Lupia  Jia Mi  Matt Wells

Undergraduate Assistants
• Maria Alyson  Carole Bryne
• Sonya Poirier  Chelsey Schwander
• Kim Rubin  Carly Toth
### LieCal Project History

<table>
<thead>
<tr>
<th>6th</th>
<th>7th</th>
<th>8th</th>
<th>9th</th>
<th>10th</th>
<th>11th</th>
<th>12th</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>2005-2009</td>
<td></td>
<td></td>
<td>2010-2012</td>
</tr>
</tbody>
</table>
Purposes

- A profile of the intended treatment of algebra in the CMP curriculum with a contrasting profile of the intended treatment of algebra in the non-CMP curricula;

- A profile of classroom experiences that CMP students and teachers have, with a contrasting profile of experiences in non-CMP classrooms; and

- A profile of student performance resulting from the use of the CMP curriculum, with a contrasting profile of student performance resulting from the use of non-CMP curricula.
Research Site

- A Larger Urban School District
- 51 schools in the district have students in the middle grades: 27 use CMP and 24 use non-CMP
Research Site (cont.)

- Diverse student population:
  - 62% African Americans
  - 21% Hispanic,
  - 12% white,
  - 4% Asian, and
  - 1% Native Americans
# Profile of Schools

<table>
<thead>
<tr>
<th>Achievement Level</th>
<th>CMP</th>
<th>Non-CMP</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Achieving</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Average Achieving</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Low Achieving</td>
<td>2</td>
<td>2</td>
</tr>
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</table>
A sample problem in CMP curriculum

The graph below shows the numbers of cans of soft drink purchased each hour from school’s vending machine in one day (6 means the time from 5:00 to 6:00, 7 represents the time from 6:00 to 7:00, and so on).

a. The graph shows the relationship between two variables. **What are the variables?**

b. Describe how the number of cans sold **changed** during the day. Give an explanation for why these changes might have occurred.
Sample problems in a US Non-CMP curriculum

Evaluate algebraic expressions:
(1) Evaluate $16 + b$ if $b = 25$.
(2) Evaluate $x - y$ if $x = 64$ and $y = 27$

Identify the solution of an equation:
$9 + w = 17$; choose one from 7, 8, 9
How is variable defined?

• “A variable is a quantity that changes or varies.”

  (CMP)

• “A variable is a symbol, usually a letter, used to represent a number.”

  (Non-CMP)
How is equation defined?

• Rather than seeing equations simply as objects to manipulate, students are shown that equations often describe relationships between varying quantities that arise from meaningful, contextualized situations. (*CMP*)

• “...a sentence that contains an equals sign, =” (*Non-CMP*)
% of Problems Involving Linear Equations

<table>
<thead>
<tr>
<th>Types of Problems</th>
<th>1equ 1va (x+2=5x)</th>
<th>1equ 2va (y=3x +4)</th>
<th>2equ 2va (3y=x+2) &amp; (y=5x+9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMP</td>
<td>5.72</td>
<td>93.03</td>
<td>1.24</td>
</tr>
<tr>
<td>Non-CMP</td>
<td>86.19</td>
<td>11.67</td>
<td>2.14</td>
</tr>
</tbody>
</table>
Mathematical Problem Posing

Reform Ideas

Curriculum

Teaching

Students’ Learning

Teachers

Problem Posing
# Number of PP tasks in Different Grade Levels

<table>
<thead>
<tr>
<th>Grade</th>
<th>Chinese – BNU</th>
<th>US -- Investigations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total tasks</td>
<td>% PP</td>
</tr>
<tr>
<td>1</td>
<td>570</td>
<td>5.96</td>
</tr>
<tr>
<td>2</td>
<td>549</td>
<td>5.65</td>
</tr>
<tr>
<td>3</td>
<td>541</td>
<td>2.77</td>
</tr>
<tr>
<td>4</td>
<td>561</td>
<td>2.85</td>
</tr>
<tr>
<td>5</td>
<td>619</td>
<td>2.91</td>
</tr>
<tr>
<td>6</td>
<td>545</td>
<td>3.12</td>
</tr>
<tr>
<td>Total</td>
<td>3,385</td>
<td>3.87</td>
</tr>
</tbody>
</table>

[1] The Investigations series does not have Grade-6 textbooks.
## Distribution of PP tasks in different content areas

<table>
<thead>
<tr>
<th>Content Area</th>
<th>Chinese (n=131)</th>
<th>US (n=60)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numbers and Operations</td>
<td>61.07</td>
<td>78.33</td>
</tr>
<tr>
<td>Algebra</td>
<td>3.05</td>
<td>18.33</td>
</tr>
<tr>
<td>Geometry</td>
<td>2.29</td>
<td>0</td>
</tr>
<tr>
<td>Measurement</td>
<td>2.29</td>
<td>0</td>
</tr>
<tr>
<td>Data analysis and probability</td>
<td>12.98</td>
<td>3.33</td>
</tr>
<tr>
<td>Undetermined</td>
<td>18.32</td>
<td>0</td>
</tr>
</tbody>
</table>
Observations

Background Information

• 50 sixth-grade classrooms
• 4 observations per classroom (2F, 2Sp)
• 2 trained observers (experienced math teachers) did the observations
• 3 reliability checks done during the year
Observation Instrument

Main Components

- Conceptual Emphases;
- Procedural Emphases;
- **Instructional Tasks**;
- Homework Problems
### Factor 1: Emphasis on Conceptual Understanding

<table>
<thead>
<tr>
<th></th>
<th>Grade 6</th>
<th>Grade 7</th>
<th>Grade 8</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CMP: n=100;</td>
<td>CMP: n=105;</td>
<td>CMP: n=112;</td>
<td>CMP: n=317</td>
</tr>
<tr>
<td></td>
<td>Non-CMP: n=95</td>
<td>Non-CMP: n=103</td>
<td>Non-CMP: n=100</td>
<td>Non-CMP: n=298</td>
</tr>
<tr>
<td>CMP</td>
<td>17.99 (4.56)</td>
<td>15.68 (4.34)</td>
<td>16.88 (4.65)</td>
<td>16.83 (4.60)</td>
</tr>
<tr>
<td>Non-CMP</td>
<td>12.33 (3.13)</td>
<td>13.60 (3.04)</td>
<td>14.12 (3.71)</td>
<td>13.37 (3.38)</td>
</tr>
<tr>
<td>ANOVA:</td>
<td>F (3, 611)=39.09, p&lt;.0001.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Factor 2: Emphasis on Procedural Knowledge

<table>
<thead>
<tr>
<th></th>
<th>Grade 6</th>
<th>Grade 7</th>
<th>Grade 8</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CMP: n=100; Non-CMP: n=95</td>
<td>CMP: n=105; Non-CMP: n=103</td>
<td>CMP: n=112; Non-CMP: n=100</td>
<td>CMP: n=317; Non-CMP: n=298</td>
</tr>
<tr>
<td>CMP</td>
<td>14.70 (3.66)</td>
<td>14.41 (3.72)</td>
<td>15.25 (4.18)</td>
<td>14.80 (3.88)</td>
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<tr>
<td>Non-CMP</td>
<td>17.16 (4.41)</td>
<td>17.72 (4.12)</td>
<td>18.33 (3.97)</td>
<td>17.75 (4.18)</td>
</tr>
<tr>
<td>T-Test</td>
<td>$P&lt;.0001$</td>
<td>$P&lt;.0001$</td>
<td>$P&lt;.0001$</td>
<td>$p&lt;.0001$</td>
</tr>
<tr>
<td>ANOVA:</td>
<td>$F(3, 611)=29.38, p&lt;.0001$.</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>
Instructional tasks were categorized into four increasingly demanding levels of cognition (Stein et al., 1996):

- Memorization;
- Procedures without connections;
- Procedures with connections; and
- Doing mathematics.
The Cognitive Level of the Instructional tasks Implemented

The Percentage Distributions of the Cognitive Demand of the Instructional Tasks

- Memorization
  - CMP (n=623): 2.09%
  - Non-CMP (n=695): 10.94%
- Procedure Without Connections
  - CMP (n=623): 53.13%
  - Non-CMP (n=695): 78.71%
- Procedure With Connections
  - CMP (n=623): 34.03%
  - Non-CMP (n=695): 9.06%
- Doing Mathematics
  - CMP (n=623): 10.75%
  - Non-CMP (n=695): 1.29%
## Student Assessment: Time Table

<table>
<thead>
<tr>
<th>Assessments</th>
<th>Fall (05-06)</th>
<th>Spring (05-06)</th>
<th>Fall (06-07)</th>
<th>Spring (06-07)</th>
<th>Fall (07-08)</th>
<th>Spring (07-08)</th>
</tr>
</thead>
<tbody>
<tr>
<td>State Tests (math &amp; reading)</td>
<td>All students</td>
<td>All students</td>
<td>All students</td>
<td>All students</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Project-Administered Test</td>
<td>6&lt;sup&gt;th&lt;/sup&gt; grade students</td>
<td>6&lt;sup&gt;th&lt;/sup&gt; grade students</td>
<td>7&lt;sup&gt;th&lt;/sup&gt; grade students</td>
<td></td>
<td>8&lt;sup&gt;th&lt;/sup&gt; grade students</td>
<td></td>
</tr>
<tr>
<td>(multiple-choice items)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Project-Administered Test</td>
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<td>7&lt;sup&gt;th&lt;/sup&gt; grade students</td>
<td></td>
<td>8&lt;sup&gt;th&lt;/sup&gt; grade students</td>
<td></td>
</tr>
<tr>
<td>(open-ended items)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Project-Administered Student Assessment Components

Assessment Task

Open-ended
13 Different Items

Multiple Choice
32 Items In each Administration

Fall'05
6 Items

Spring'06
3 Forms
5 Items for Each

Spring'07
3 Forms
5 Items for Each

Spring'08
3 Forms
5 Item for Each
Achievement Scaled Scores

• A scaled score is a generic term for a mathematically transformed student raw score on an assessment.

• Using scaled scores, rather than raw scores, assessment results can be placed on the same scale even though students responded to different tasks and at different times.

• The two-parameter Item Response Theory (IRT) model was used to scale student assessment data.
Achievement Scaled Scores

• The two parameters are: An item difficulty index (easy or hard item) and an item discrimination index (how well an item distinguishes lower from higher achievers).

• Using the two-parameter IRT model, student responses were scaled across all forms and three assessment times.
Quantitative Analysis of student achievement data

- Repeated Measures ANOVA
- ANCOVA
- HLM Growth Curve Modeling
- HLM Cross-Sectional
A Case from LieCal Project

Mean Scores for CMP and non-CMP Students on the Open-ended Tasks
<table>
<thead>
<tr>
<th>LieCal Project History</th>
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<tbody>
<tr>
<td>6th</td>
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<td>2005-2009</td>
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<td>2010-2012</td>
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# Data Source

<table>
<thead>
<tr>
<th>Assessments</th>
<th>Fall 6th grade</th>
<th>Spring 6th grade</th>
<th>Fall 7th grade</th>
<th>Spring 7th grade</th>
<th>Fall 8th grade</th>
<th>Spring 8th grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>State Tests (math &amp; reading)</td>
<td>All students</td>
<td>All students</td>
<td></td>
<td>All students</td>
<td></td>
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</tr>
<tr>
<td>Project-Administered Test (multiple-choice items)</td>
<td>6th grade students</td>
<td>6th grade students</td>
<td></td>
<td>7th grade students</td>
<td></td>
<td>8th grade students</td>
</tr>
<tr>
<td>Project-Administered Test (open-ended items)</td>
<td>6th grade students</td>
<td>6th grade students</td>
<td></td>
<td>7th grade students</td>
<td></td>
<td>8th grade students</td>
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# Data Source

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<th>10&lt;sup&gt;th&lt;/sup&gt; Grade</th>
<th>11&lt;sup&gt;th&lt;/sup&gt; Grade</th>
<th>12&lt;sup&gt;th&lt;/sup&gt; Grade</th>
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<tbody>
<tr>
<td>Open-ended tasks</td>
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<td></td>
<td>√</td>
<td>√</td>
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<tr>
<td>Multiple choice tasks</td>
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<tr>
<td>State Test Data</td>
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<tr>
<td>Mathematics Grades</td>
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<td>√</td>
<td>√</td>
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<td>Enrollment in Advanced Math Courses</td>
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<td>SAT/ACT Registration and Scores</td>
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<td>√</td>
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<td>Covariate(s)</td>
<td>F-Value</td>
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<tr>
<td>--------------------------------------------------</td>
<td>---------</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>PI-developed 6th grade MC tasks</td>
<td>5.13*</td>
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<tr>
<td>PI-developed 6th grade OE tasks</td>
<td>3.90*</td>
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<td>6th grade State math test scaled score</td>
<td>9.58**</td>
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<tr>
<td>7th grade State math test scaled score</td>
<td>9.57**</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>8th grade State math test scaled score</td>
<td>11.79***</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Problem Posing and PS Strategies

• Posing similar or more complex problems

• More Abstract strategies
Research in Medical Education

• PBL v.s. Lecture
• Immediate assessment
Knowledge: Lecture > PBL
Clinical: PBL > Lecture
• Delayed assessment
Knowledge: PBL > Lecture
Clinical: PBL > Lecture
Some Research Findings
(Cai & Merlino, 2011)

- A total of 1316 high school students
- Different programs:
  - 285 Non-college preparation mathematics
  - 858 college preparation math (traditional)
  - 173 college preparation math (NSF-Funded)
Survey Instrument

We are interested in learning how you think and feel about mathematics. Please take a few minutes to think about the following questions and write how you truly feel. There are no right or wrong answers.

- If Math were a **food**, it would be because_____
  ________________________________

- If Math were a **color**, it would be because_____
  ________________________________

- If Math were an **animal**, it would be because
To show they like mathematics

• “Purple is my favorite color. It’s my birth stone color plus it brings passionate. That’s how I feel about math.”
To show they like mathematics (Why?)

- “Math is like steak because math is a full, expansive subject. However, like a steak there are tough bits of gristle scattered throughout obstacles you must work around. The full meal is satisfying, but the process of eating is somewhat unusually strenuous.”

- “Vegetables are good for you, and so is mathematics for daily things. It is needed in life. Some people like it, and some people don’t, but you still need it to live a healthy life.”
To show they dislike mathematics

- “I would say a mosquito, because whatever you do to try and get away from it, it always comes back. It’s annoying because you hate taking math every year, and whatever you try to do to stop it, it always fails.”
To show they dislike math (Why?)

- “It is like gum. You chew gum and use it to freshen up your breath, but in the end, it’s worthless and doesn’t have any nutrition or vitamins. Math is used in school to determine your intelligence, but there is no need for it later.”
Analyses of Responses

- **Quantitative Analysis:** Holistic scoring (1 - 5)
  - 1 Point: Very Negative
  - 2 points: Moderately Negative
  - 3 points: Neutral or Ambivalent
  - 4 points: Moderately Positive
  - 5 points: Very Positive.

- **Qualitative Analysis:** Reveal what kinds of metaphors students used and why
Figure 2. Percentage Distribution at Each Attitude Level
Background Information in the Ten School Districts in GPSMP (Kramer, Cai, & Merlino, in press)

<table>
<thead>
<tr>
<th>School District</th>
<th>Curriculum (20 Middle School)</th>
<th>Approximate # of Students (Middle School)</th>
<th>Curriculum (High School)</th>
<th>Approximate # of Students (12 High School)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 District A (PA)</td>
<td>CMP</td>
<td>4000</td>
<td>CPMP</td>
<td>5000</td>
</tr>
<tr>
<td>2 District B (PA)</td>
<td>MiC</td>
<td>2000</td>
<td>CPMP</td>
<td>2000</td>
</tr>
<tr>
<td>3 District C (PA)</td>
<td>MiC</td>
<td>1000</td>
<td>IMP</td>
<td>2000</td>
</tr>
<tr>
<td>4 District D (PA)</td>
<td>MiC</td>
<td>1000</td>
<td>IMP</td>
<td>2000</td>
</tr>
<tr>
<td>5 District E (NJ)</td>
<td>CMP</td>
<td>500</td>
<td>CPMP</td>
<td>500</td>
</tr>
<tr>
<td>6 District F (PA)</td>
<td>MiC</td>
<td>1000</td>
<td>IMP</td>
<td>1000</td>
</tr>
<tr>
<td>7 District G (NJ)</td>
<td>CMP</td>
<td>1000</td>
<td>CPMP</td>
<td>1000</td>
</tr>
<tr>
<td>8 District H (NJ)</td>
<td>CMP</td>
<td>1000</td>
<td>IMP</td>
<td>2000</td>
</tr>
<tr>
<td>9 District I (PA)</td>
<td>CMP</td>
<td>1000</td>
<td>IMP</td>
<td>2000</td>
</tr>
<tr>
<td>10 District J (PA)</td>
<td>CMP</td>
<td>1000</td>
<td>CPMP</td>
<td>2000</td>
</tr>
</tbody>
</table>
Four Factors for “Will to Reform”

• Superintendent support for the reform program;
• Principal support for the reform program;
• Teacher “buy-in” to the reform program;
• Coherence of School District support for the reform program
Scatter-plot of “Treatment Growth” (zmath04-zmath98 in PA; zmath04-zmath99 in NJ)
Scatter-plot of “Treatment Growth” - “Control Growth”
Outline

• Three Reflections
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• LieCal Project
• Mathematical Model of Marriage
The Marriage Equation: A practical theory for predicting divorce & scientifically-based marital therapy

John Gottman and James D. Murray
Gathering a Couple’s Data

Video is taken of the couple discussing a topic of contention, such as money, sex, housing, in-laws etc.

An accepted scoring system assigns a specific number (positive or negative) to each statement.

The scores (positive – negative) for the husband (H) and the wife (W) for each turn of speech (t) are plotted as functions of time. It measures the average positivity of each spouse as a function of time (t).
Data Representation: Typical Data for Low Risk Couple

Cumulative “positive-negative” scores for each turn of speech for the husband and wife.

Examples:
- affection +4
- disgust -3
- whining -1
- contempt -4
Typical High Risk Couple’s Interaction

High Risk Couple

Unstable marriage

0.8 to 1 positive to negative ratio
Mathematical Model of the interaction

Wife’s score at time $t + 1$
\[
W_{t+1} = a + r_1 W_t + I_{HW}(H_t)
\]

Husband’s score at time $t + 1$
\[
H_{t+1} = b + r_2 H_t + I_{WH}(W_t)
\]
Basic Marriage Types

Observations of couples (RCISS - Rapid Couples Interacting Scoring System) and mathematical model 5 types of marriages:

3 stable: (1) Volatiles, (2) Validators, (3) Avoiders
2 unstable: (1) Hostiles, (2) Hostile-Detached
Pre-Marital Therapy

Husband and Wife tend toward negativity
Post-Marital Therapy

Husband and Wife become more positive

- Wife's Cumulative Score
- Husband's Cumulative Score
!!! Thank You !!! and Questions!

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