BILINGUAL TEACHERS’ PROFESSIONAL NOTICING OF BILINGUAL CHILDREN’S MATHEMATICAL THINKING

MSU Mathematics Education Colloquium

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TEXAS STATE UNIVERSITY
SAN MARCOS
The rising STAR of Texas
Introductions: Important numbers about me

• 1,349
  – The number of miles I traveled from Austin, TX

• $4 \times 4$ or $2 + 2 + 4 + 4+4$
  – The number of feet/paws in my house

• 3 tens and 15 ones
  – The number of elementary mathematics methods courses I’ve taught over the course of 9 years

What is numerically interesting about you?
Overview

- Noticing – An introduction
- Three important areas of study
  - Children’s Mathematical Thinking
  - Professional Noticing of Children’s Mathematical Thinking
  - Emergent Bilingual Students: Teaching and Learning of Mathematics
- Research Study
  - Data Analysis
  - Findings
  - My Wonderings: Going Beyond “Professional Noticing of Children’s Mathematical Thinking”
  - Dialogue (Q&A)
From Merriam-Webster:

2 notice /ˈnəʊtɪs/ verb

notices; noticed; noticing

Learner's definition of NOTICE

[+ object]
: to become aware of (something or someone) by seeing, hearing, etc.

- He noticed his friend sitting at the next table. = He noticed that his friend was sitting at the next table.
- She noticed a smell of gas.
- You didn't notice that I got my hair cut.
- She noticed me leaving the meeting early.
- The police noticed [=observed] a connection between the murders.
- I noticed an error in the book.
- The problem was first noticed several days ago.
- I couldn’t help noticing the spot on his tie. = I couldn’t help but notice the spot on his tie. [=I saw the spot even though I wasn’t trying to look for it]

notar = to notice

Conjugation of NOTAR

Regular

<table>
<thead>
<tr>
<th></th>
<th>Present</th>
<th>Preterit (Past)</th>
<th>Imperfect</th>
</tr>
</thead>
<tbody>
<tr>
<td>yo</td>
<td>noto</td>
<td>noté</td>
<td>notaba</td>
</tr>
<tr>
<td>tú</td>
<td>notas</td>
<td>notaste</td>
<td>notabas</td>
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<td>notó</td>
<td>notaba</td>
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<td>notábamos</td>
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<td>notasteis</td>
<td>notabais</td>
</tr>
<tr>
<td>ellos/ellas/Uds.</td>
<td>notan</td>
<td>notaron</td>
<td>notaban</td>
</tr>
</tbody>
</table>
Mathematics Teacher Noticing

■ What teachers “attend” to in a classroom setting and how they “make sense” of the events that occur (Sherin, Jacobs & Philipp, 2011)

■ Noticing studies have existed since the 1960s (Erickson, 2011)

■ Now there is a specific focus on looking at what mathematics teachers notice about mathematics classroom events
What do you notice?

- Ashley (1st grader) will solve the following problem:
  - *Carla has 7 dollars. How many more dollars does she need to earn so that she will have 11 dollars to buy a puppy?*

- How might she solve it? What is the answer?
What do you notice?

Carla has 7 dollars. How many more dollars does she need to earn so that she will have 11 dollars to buy a puppy?
Characteristics of Noticing

■ Noticing is something everyone does
  - (Mason, 2002; Crespo, 2006; Sherin et al., 2011)

■ Experience can affect noticing
  - (Jacobs, Lamb & Philipp, 2010; Erickson, 2011)

■ Teachers often respond to what they notice
  - (Kazemi & Franke, 2004; Breyfogle, 2005)

■ What a teacher notices can be redirected
  - (van Es & Sherin, 2008; Sherin, Jacobs & Philipp, 2011)
Brief overview of 3 areas of study

- Children's Mathematical Thinking
- Professional Noticing of Children's Mathematical Thinking
- Emergent Bilingual Students: Teaching and Learning of Mathematics
Children’s Mathematical Thinking

- “No matter how lucidly and patiently teachers explain to their students, they cannot UNDERSTAND for their students.” (Shifter & Fosnot, 1993)

- Children come to school with a rich store of knowledge that they can use to solve mathematics problems. (Carpenter, et al., 1989;1999)
  - Cognitively Guided Instruction (CGI) studies provide a framework for understanding children’s mathematical thinking
Cognitively Guided Instruction

- Learning to Add and Subtract: An Exercise in Problem Solving
  - Carpenter, 1985
- The Acquisition of the Addition and Subtraction Concepts in Grades One Through Three
  - Carpenter & Moser, 1984
- Teachers Pedagogical Content Knowledge of Student’s Problem Solving in Elementary Arithmetic
  - Carpenter, Fennema, Peterson & Carey, 1988
- Using Knowledge of Children’s Mathematics Thinking in Classroom Teaching: An Experimental Study
  - Carpenter, Fennema, Peterson, Chiang, & Loef, 1989
Research <=> Practice
Importance of Mathematical Discussions

- Community of practice
- Centered around a task or purpose
- Relevant to strands of mathematical proficiency
- Teacher as facilitator

Maria is going to have a party and wants to buy sandwiches to eat. Each sandwich costs 6 dollars. Maria has 54 dollars. How many sandwiches can Maria buy?
Challenges

“Ambitious” mathematics teaching (Lampert, Beasley, Ghousseini, Kazemi & Franke, 2010)

“Superficial” teacher attempts at problem solving (Stein, Engle, Smith & Franke, 2008)
Professional Noticing of Children’s Mathematical Thinking

- CGI studies provide understanding the progression of children’s mathematical thinking
- Teachers who understand that progression can cultivate problem solving classrooms with specific teacher moves to help children progress their mathematical thinking
- In essence, these teachers are “noticing” something important

Jacobs, Lamb, Philipp, Randolph, Schapelle, Burke (2007)
3 noticing skills
(Jacobs, Lamb & Philipp, 2010)

- **Attending** to children’s strategies – what are the important mathematical details about children’s strategies?

- **Interpreting** children’s mathematical understandings – To what extent does a teacher use research on children’s mathematical thinking to interpret and reason about a child’s strategy?

- **Deciding How to Respond** on the basis of children’s understandings – To what extent does a teacher use what they know about research on children’s mathematical thinking to reason about what to do next
If we are to change the way in which teachers engage children in problem solving, we must first understand what teachers notice about engaging children in problem solving.
Emergent Bilingual Students

- Labels and language are important
- Move away from terms like “English Language Learner”
  - Instead of seeing students as “limited” or “Learners of English” educators can begin to see them through their developing bilingualism (Garcia & Kleifgen, 2010)
  - “When officials and educators ignore the bilingualism that these students can and must develop through schooling in the United States, they perpetuate inequities in the education of these children. That is, they discount the home languages and cultural understandings of these children and assume that their educational needs are the same as a monolingual child.” (pg. 2)
Emergent Bilingual Students

- Majority of EL students speak Spanish (76% of elementary ELL students) (IDRA, 2009)
- Instructional programs for ELs vary across states, cities and even districts
- Many teachers do not feel prepared to teach ELs (Lucas et al., 2008)
- Many teachers do not have the linguistic training or professional development opportunities to learn about teaching mathematics to emergent bilingual students (Aguirre & Bunch, 2012)
  - Language Demands: Reading, Listening, Writing, Speaking and Representing
Dispelling the myths

- Children learn languages easier/faster than adults
  - *Adults have more resources to draw upon due to their English language proficiency*

- Teachers should not allow students to use their native language as it will slow their English language development
  - *Allowing students to use their native language facilitates cognitive and academic growth*

- Math should be easier for non-English speakers because mathematics is universal
  - *Language plays an important role in learning mathematics*
Language Considerations

How many different ways can you say: 75 – 23?
75 minus 23
75 less 23
75 decreased by 23
23 less than 75

What are some homonyms?
Difference, even, figure, left

What are some homophones?
Right, write; some, sum; cent, sent; one, won

Focusing on cognates can help!
Algoritmo, centro, doble, equivalente
Guiding Principles for Teaching Mathematics to English Language Learners

- Ramirez & Celedon-Pattichis (2012)
  - Challenging mathematical tasks
  - Linguistically sensitive social environment
  - Support for learning English while learning mathematics
  - Mathematical tools and modeling as resources
  - Cultural and linguistic differences as intellectual resources
**Table 2**

**Levels of Engagement with Children’s Mathematical Thinking**

*Level 1:* The teacher does not believe that the students in his or her classroom can solve problems unless they have been taught how.
- Does not provide opportunities for solving problems.
- Does not ask the children how they solved problems.
- Does not use children’s mathematical thinking in making instructional decisions.

*Level 2:* A shift occurs as the teachers begin to view children as bringing mathematical knowledge to learning situations.
- Believes that children can solve problems without being explicitly taught a strategy.
- Talks about the value of a variety of solutions and expands the types of problems they use.
- Is inconsistent in beliefs and practices related to showing children how to solve problems.
- Issues other than children’s thinking drive the selection of problems and activities.

*Level 3:* The teacher believes it is beneficial for children to solve problems in their own ways because their own ways make more sense to them and the teachers want the children to understand what they are doing.
- Provides a variety of different problems for children to solve.
- Provides an opportunity for the children to discuss their solutions.
- Listens to the children talk about their thinking.

*Level 4A:* The teacher believes that children’s mathematical thinking should determine the evolution of the curriculum and the ways in which the teachers individually interact with the students.
- Provides opportunities for children to solve problems and elicits their thinking.
- Describes in detail individual children’s mathematical thinking.
- Uses knowledge of thinking of children as a group to make instructional decisions.

*Level 4B:* The teacher knows how what an individual child knows fits in with how children's mathematical understanding develops.
- Creates opportunities to build on children’s mathematical thinking.
- Describes in detail individual children’s mathematical thinking.
- Uses what he or she learns about individual students’ mathematical thinking to drive instruction.
The Study

- Southern U.S.A. elementary school
  - About 750 students in PreK- 5th grade
  - 95% qualified for free lunch
  - 90% Latino
  - 59% “limited English proficient” (*emergent bilingual)
  - Followed transitional model of bilingual education

- Three 3rd grade bilingual teachers selected five bilingual students to form weekly problem solving groups that met for 10 weeks
  - Students were selected based on mathematical levels and English proficiency levels

- Teacher Study Groups (TSGs) (Crespo, 2006) met five times to review video from problem solving groups and discuss student work

- I designed weekly problems based on teacher input
Research Questions

- What do bilingual teachers notice about engaging bilingual children in problem solving in small groups?

- What do teachers notice about their children’s English language acquisition and the learning of mathematics when facilitating small group discussions as they engage children in discussions of problem solving?
Data Analysis

- Transcriptions of Teacher Study Groups, Small Problem solving Groups
- Pre and post study teacher interviews, student problem solving interviews, teacher written reflections
- Looked for evidence of focus on children’s mathematical thinking
- Jacobs et al. (2010) framework for coding of 3 noticing skills
- I was both researcher and participant (Merriam, 1998)
- Episodes to code were instances in which teachers were reflecting on or discussing engaging children in problem solving
Focusing questions for Teacher Study Groups

- What did students do to solve the problems?
- How do you know (for sure) what it is that a student understands?
- Observations about questions that were asked to students
- Observations about interactions/conversations between teacher and students
- Observations about the role of language when facilitating a discussion
Findings

- Instances of 3 types of Noticings and Other Concerns

<table>
<thead>
<tr>
<th>Teacher Study Groups</th>
<th>Attending to children’s strategies</th>
<th>Interpreting children’s mathematical understandings</th>
<th>Deciding how to respond</th>
<th>Other Concerns</th>
</tr>
</thead>
<tbody>
<tr>
<td>TSG 2</td>
<td>22</td>
<td>26</td>
<td>6</td>
<td>11</td>
</tr>
<tr>
<td>TSG 3</td>
<td>26</td>
<td>37</td>
<td>11</td>
<td>33</td>
</tr>
<tr>
<td>TSG 4</td>
<td>5</td>
<td>15</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>TSG 5</td>
<td>13</td>
<td>33</td>
<td>12</td>
<td>28</td>
</tr>
<tr>
<td>TSG 6</td>
<td>21</td>
<td>39</td>
<td>8</td>
<td>21</td>
</tr>
<tr>
<td>Total</td>
<td>87</td>
<td>150</td>
<td>38</td>
<td>102</td>
</tr>
</tbody>
</table>
### Findings – Artifacts of Practice that assisted with noticing

<table>
<thead>
<tr>
<th>Type of Artifact of Practice</th>
<th>Attending to children’s strategies</th>
<th>Interpreting children’s mathematical understandings</th>
<th>Deciding how to respond</th>
<th>Other Concerns</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recall</td>
<td>11</td>
<td>35</td>
<td>7</td>
<td>40</td>
<td>93 (25%)</td>
</tr>
<tr>
<td>Student Notebook</td>
<td>54</td>
<td>62</td>
<td>11</td>
<td>18</td>
<td>145 (38%)</td>
</tr>
<tr>
<td>Transcript of small group session</td>
<td>8</td>
<td>12</td>
<td>11</td>
<td>18</td>
<td>50 (13%)</td>
</tr>
<tr>
<td>Video of small group session</td>
<td>4</td>
<td>27</td>
<td>9</td>
<td>22</td>
<td>62 (16%)</td>
</tr>
<tr>
<td>Transcript and student notebook</td>
<td>8</td>
<td>11</td>
<td>0</td>
<td>3</td>
<td>22 (6%)</td>
</tr>
<tr>
<td>Transcript and video</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>6 (2%)</td>
</tr>
</tbody>
</table>

| Total                       | 88                               | 150                                              | 38                      | 102           | 378 (100%) |


<table>
<thead>
<tr>
<th>Instances of Attending to Children’s Strategies</th>
<th>Definition</th>
<th>Number of Instances</th>
<th>Percent of Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic description of student strategy</td>
<td>Comments tend toward general features of the strategy</td>
<td>43</td>
<td>50%</td>
</tr>
<tr>
<td>Detailed description of student strategy</td>
<td>Comments include details such as how the child counted or decomposed numbers.</td>
<td>12</td>
<td>13%</td>
</tr>
<tr>
<td>Correctness of Answer</td>
<td>Comments focus on correctness of student strategy with little to no other descriptors</td>
<td>5</td>
<td>6%</td>
</tr>
<tr>
<td>Unclear on Student Strategy</td>
<td>Comments reflect on not understanding what a student did or not being sure why a student did a specific strategy</td>
<td>23</td>
<td>26%</td>
</tr>
<tr>
<td>Basic description of student explanation</td>
<td>Comments tend to repeat the answers students without any supporting comments as to strategies used</td>
<td>1</td>
<td>1%</td>
</tr>
<tr>
<td>Detailed description of student explanation</td>
<td>Comments include details of student’s explanations to each other</td>
<td>4</td>
<td>5%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>88</strong></td>
<td><strong>100%</strong></td>
</tr>
</tbody>
</table>
Example: Attending

**Figure 4.1 Samuel’s Strategy for Doughnuts Problem**
For a party, the third grade class has 3 boxes of doughnuts with 10 doughnuts in each box, and they also have 18 individual doughnuts. How many doughnuts do they have altogether?

Teacher said: He used base ten blocks and knew how to use the algorithm.
## Interpreting

<table>
<thead>
<tr>
<th>Types of Interpretations</th>
<th>Characteristics</th>
<th>Number of Instances</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evaluative Interpretations</td>
<td>Comments about what children understand are not based on children’s strategies but instead extrapolations for teachers’ interpretations on why children are doing what they do</td>
<td>49 (33%)</td>
</tr>
<tr>
<td>Interpretations of Strategies by Individual Students</td>
<td>Comments are based on student’s strategies or mathematical understandings of children’s problem solving learning, three levels of evidence.</td>
<td>78 (52%)</td>
</tr>
<tr>
<td>Interpretations of Small Group Discussions</td>
<td>Comments are based on children’s understandings in the course of discussing students’ problem solving strategies, three levels of evidence.</td>
<td>23 (15%)</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>150 (100%)</td>
</tr>
</tbody>
</table>
Deciding how to respond

<table>
<thead>
<tr>
<th>Categories of Deciding How to Respond</th>
<th>Number of Instances</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher move based on other teachers’ small groups</td>
<td>4</td>
</tr>
<tr>
<td>Specific activity/teacher move to help struggling student</td>
<td>16</td>
</tr>
<tr>
<td>Teacher move to facilitate discussion in small group</td>
<td>8</td>
</tr>
<tr>
<td>Teacher move to guide student toward more efficient strategies</td>
<td>10</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>38</strong></td>
</tr>
</tbody>
</table>
## Other concerns

<table>
<thead>
<tr>
<th>Types of Other Concerns</th>
<th>Examples of Noticing</th>
<th>Number of Instances</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working one on one with a student</td>
<td>Issues of time management, how to help struggling students, how directive should teachers be, what to do with struggling students</td>
<td>36</td>
</tr>
<tr>
<td>Working with a group of students</td>
<td>Issues of time management, interpreting when/if discussion is beneficial, management of different mathematical levels, understanding priorities and procedures of routines of practice</td>
<td>29</td>
</tr>
<tr>
<td>Feelings about doing mathematics</td>
<td>Comments about how students feel or interpret feelings about doing mathematics</td>
<td>6</td>
</tr>
<tr>
<td>What happens if? scenarios</td>
<td>Time management, managing student strategy sharing, unexpected student responses</td>
<td>11</td>
</tr>
<tr>
<td>How am I doing?</td>
<td>Reflection on the role of the teacher as manager, facilitator, reconsideration of decisions</td>
<td>20</td>
</tr>
</tbody>
</table>

Total 102
Findings - Noticing

- For both attending and interpreting noticing skills, teachers would focus on two things
  - Incorrect and correct answers
  - Making sure the teacher understood a student’s strategy
- The attention to detail for attending was basic or interpretations provided low evidence of using understanding of children’s mathematical thinking in their reasoning about what children had done.
- Results showed not only what teachers interpreted but also what they chose to interpret
Findings - Noticing

- More than half of interpretations of student’s thinking were based on one-on-one interactions with a student
- Reflections on multiple student strategies at once was limited
- There is a clear result as to when teachers decided to respond – most often to help struggling students
- Other Concerns – What teachers seemed to take into account in reasoning about what happens when asking students to problem solve, as well as when facilitating a small group of students
Findings – English language acquisition

- Teachers noticed when students spoke in English or Spanish when the teacher was speaking in English.
- Most of the focus from the TSGs was the emerging realization about the teacher’s own comfort level with facilitating problem solving in Spanish.
- There was a concern with “fairness” of allowing students opportunities to share their thinking in English.
- The high stakes standardized test was the deciding factors for many of their language decisions for mathematics instruction.
Relevance of Findings

Previous studies did not focus on teachers working with a group of students - this study illuminates the complexities of working with a small group of students and the implications for engaging whole classes in problem solving.

Teachers need opportunities to reason through what it means to engage children in problem solving.

TSGs and looking at artifacts of practice can provide opportunities to engage in children’s mathematical thinking.

These teachers’ understandings of what it means to teach mathematics to emergent bilingual students have been shaped by the accountability system that separates language learning from mathematics learning. We need further research to help this group of teachers better meet the demands of teaching in a linguistically diverse classroom.
“I think we as bilingual teachers are stuck in this position where we always feel like we're doing the wrong thing by somebody. Like either we're hurting the kids who could be doing more in English by not doing English. Which is probably not true, like we're probably not hurting them, it's just that we're not like, really focusing on how we're developing their English. Or we're, you know, going too far, too fast in English for the kids who still need Spanish. Like, I don't know, I just always feel that way. Like I'm not doing the right thing for everybody no matter what I do. Which is unfortunate.” (Ms. Ambers, 3rd grade teacher)
Going beyond “Professional Noticing of Children’s Mathematical Thinking”

- What has not been made visible?
  - *Complexity Theory in Education* (Hoban, 2002; Davis & Sumara, 2006)
  - *Students’ Reciprocal Noticing* (Dominguez & Adams, 2013; Dominguez, in review)
    - “What is missing is a way to help teachers attend to the multiple dimensions of teaching mathematics that are based in research on children’s mathematical thinking as well as language, culture and power.” (pg. 165)

- Next steps?? Translanguaging in mathematics education
Thank you!

- Dialogue (Q & A)

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