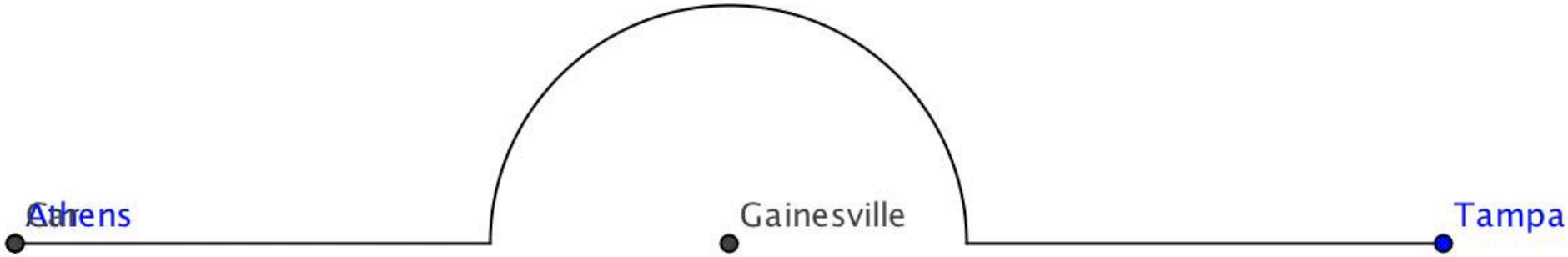
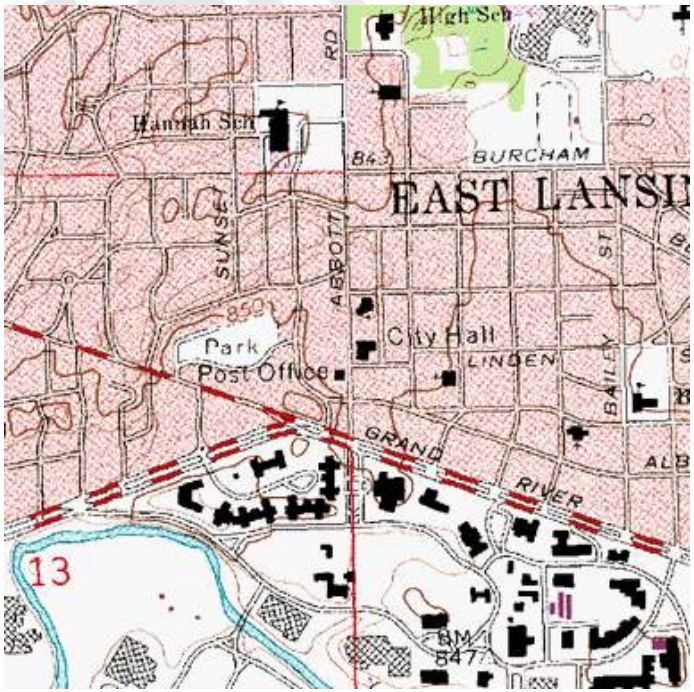
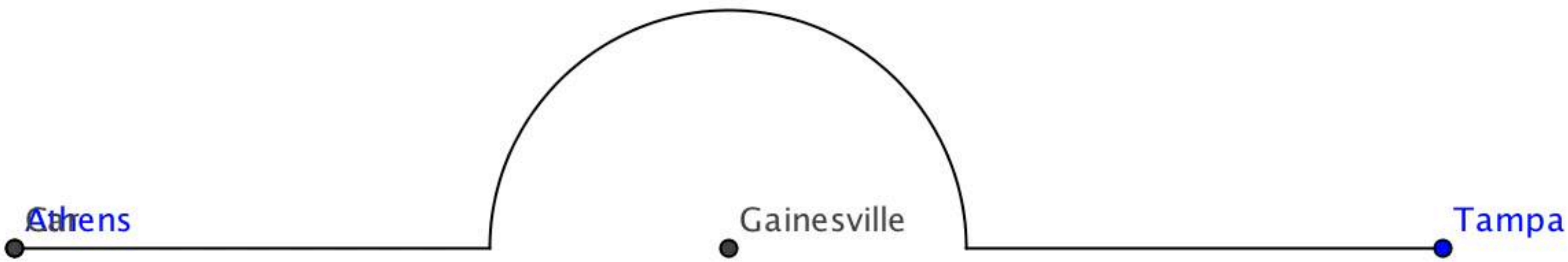


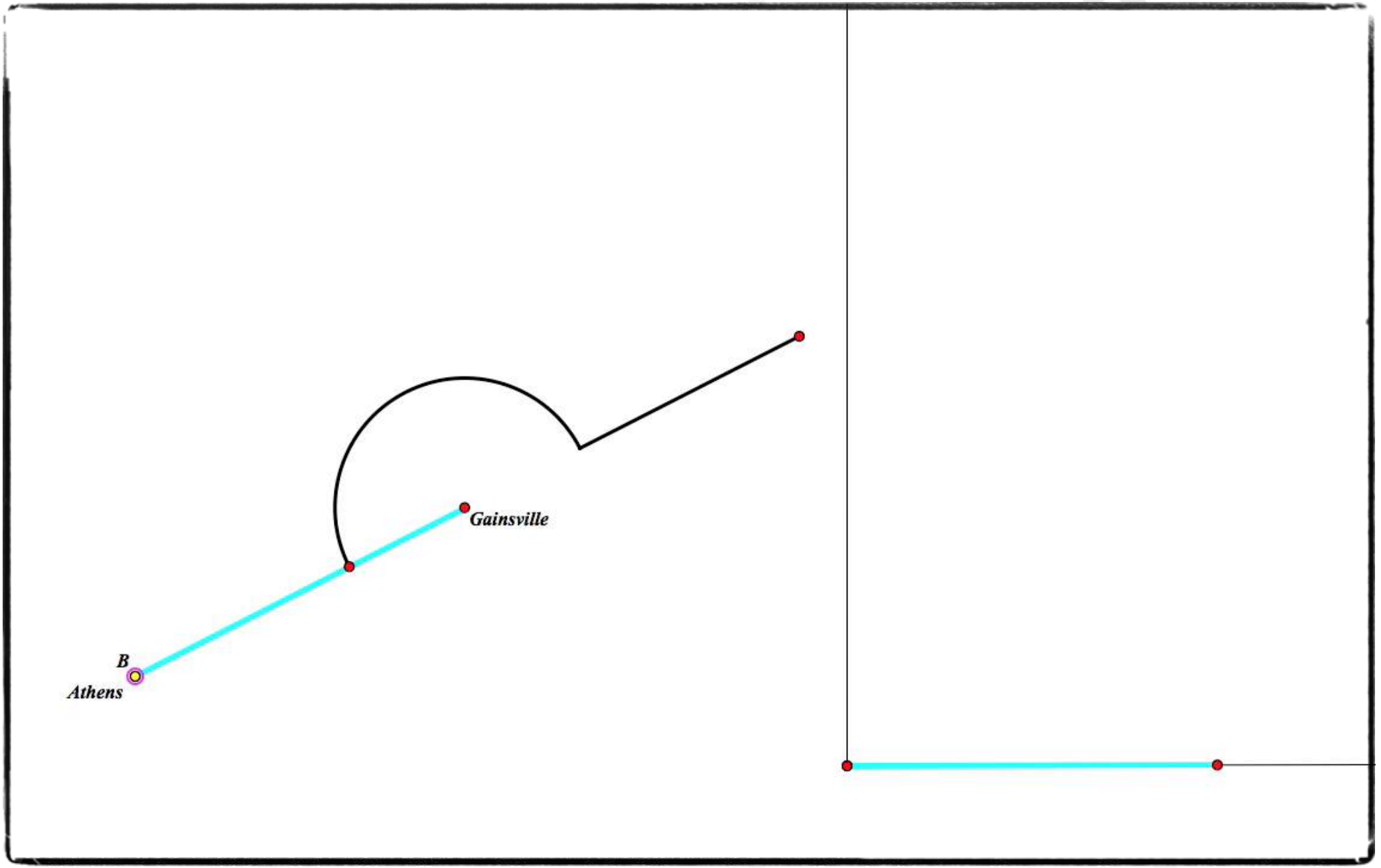
Students' Graphing Activities: Re-presentations of What?

Kevin C. Moore
University of Georgia



Sketch a graph that relates the car’s distance from Athens to the car’s distance from Gainesville.

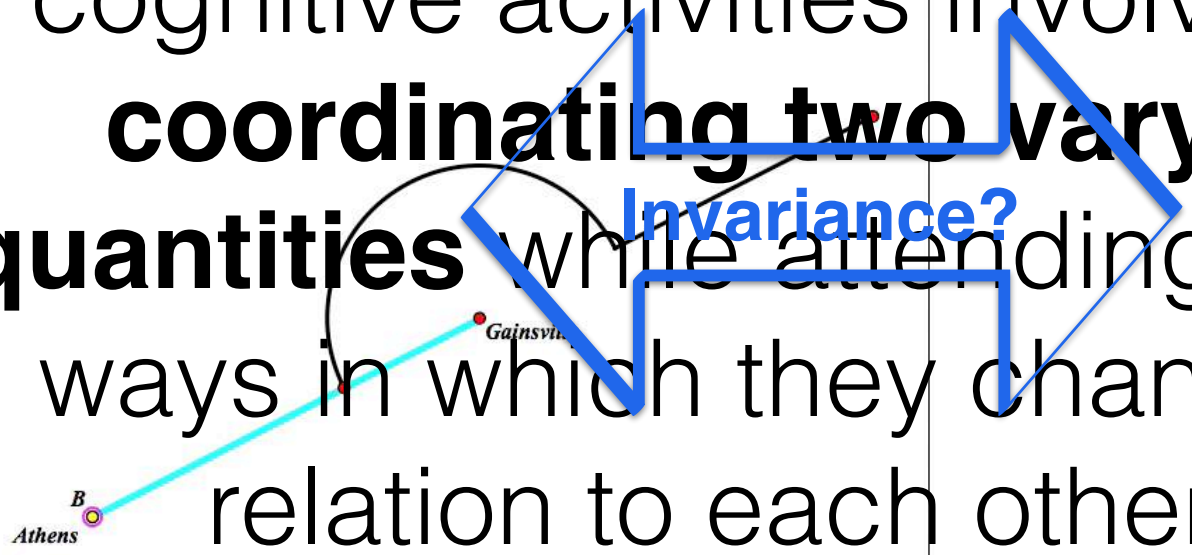



Gainsville
B
Athens

Thompson and Saldanha's (1998) Covariation

"Covariational reasoning [is] the cognitive activities involved in **coordinating two varying quantities** while attending to the ways in which they change in relation to each other."

- Carlson et al., 2002



"…conceiving of a ***multiplicative object***—an object that is produced by uniting in mind two or more quantities simultaneously."

- Thompson, 2011

Figurative Thought

Operative Thought

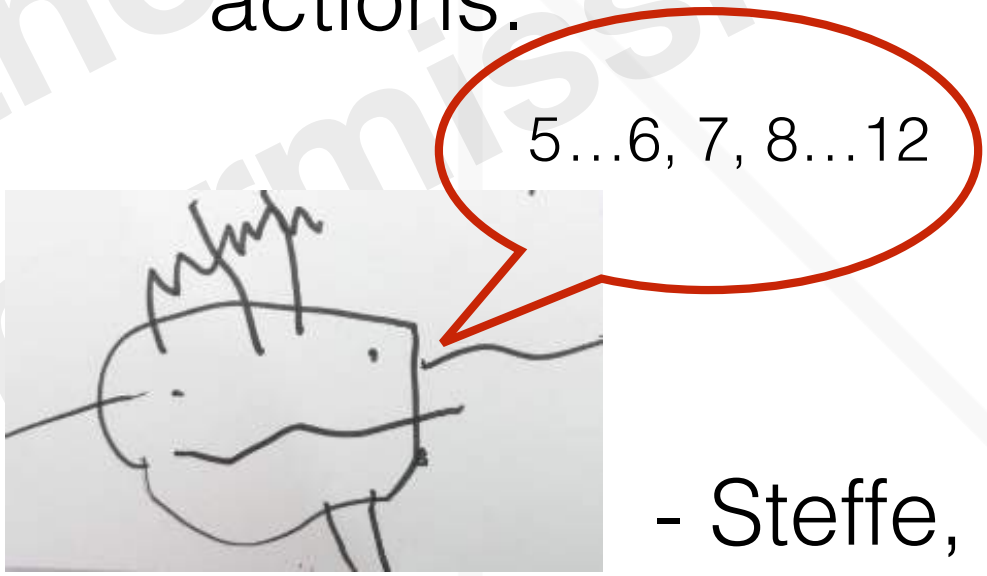
Figurative Thought (Piaget, 2001)

…based in, constrained to, and dominated by **perceptual** elements and **sensorimotor** activity (and its results).

- Steffe, 1991

Operative Thought (Piaget, 2001)

Thought that foregrounds the coordination and re-presentation of **internalized actions** (which might entail fragments of figurative thought) and the **transformation** of those actions.

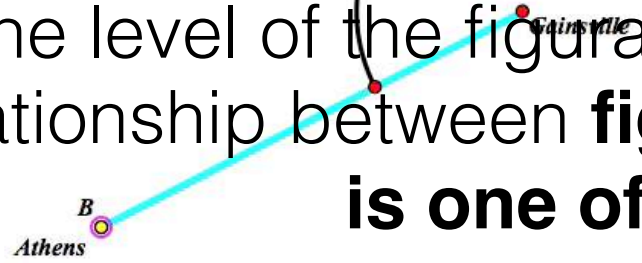


- Steffe, 1991

Figurative?

When a person's actions of thought **remain predominantly within schemata associated with a given level (of control)**, his or her thinking can be said to be **figurative** in relation to that level. When the actions of thought move to the **level of controlling schemata**, then the thinking can be said to be **operative** in relation to the level of the figurative schemata. That is to say, the relationship between **figurative and operative thought is one of figure to ground.**

- Thompson, 1985, p. 195



Operative?

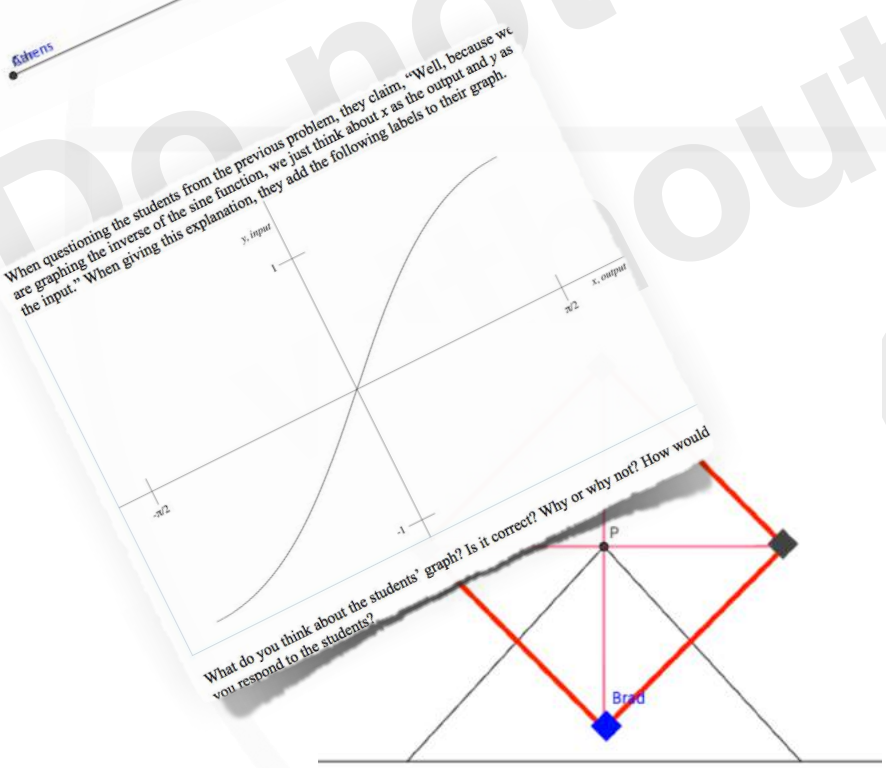
Illustrations

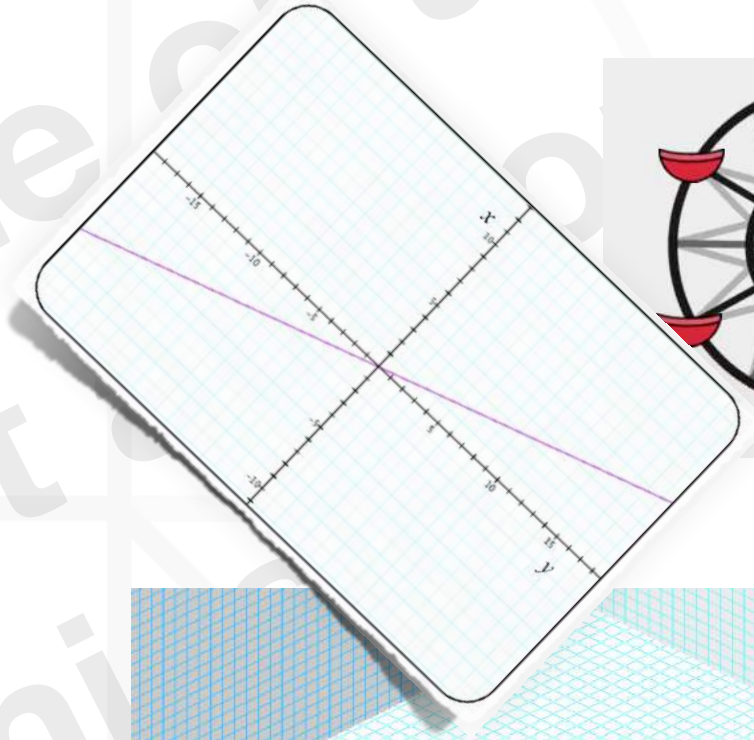
Data Sources and Epistemic Subjects

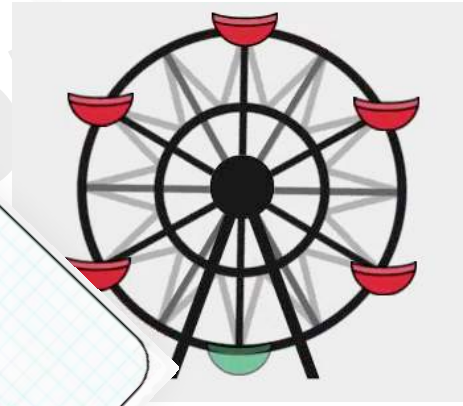
Data Sources

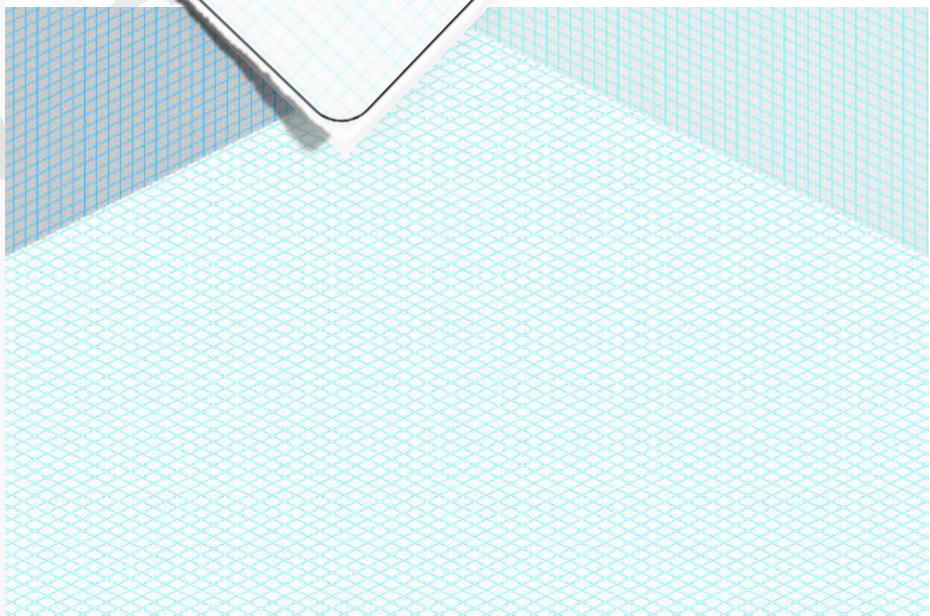
Clinical Interviews (Ginsburg, 1997)

Teaching Experiments (Seffe & Thompson, 2000)









Epistemic Subjects

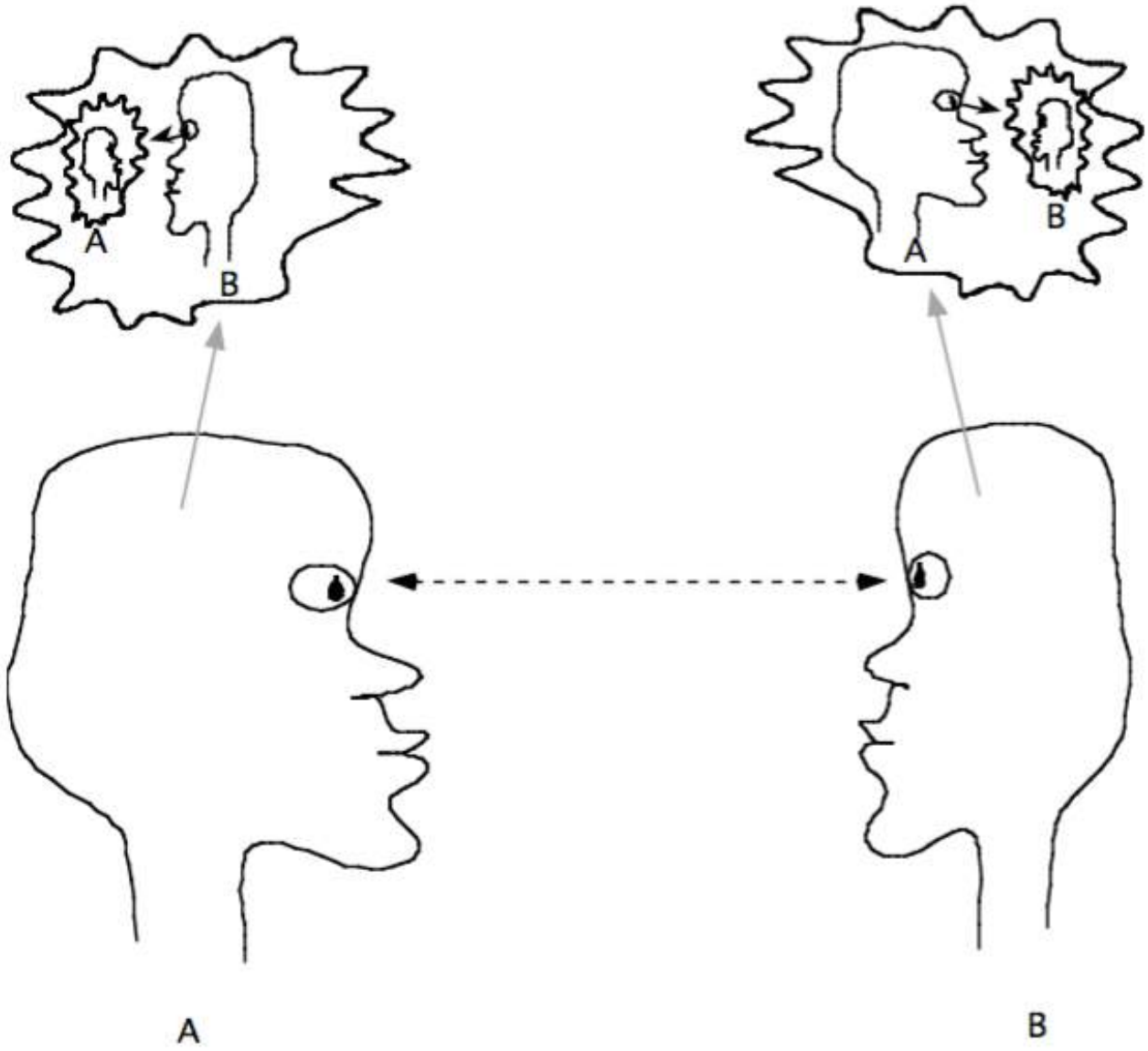
Epistemic Subjects

A characteristic of thinking that has *stabilized* within a researcher's thinking across the second-order models he or she has created for particular students' mathematical meanings (Steffe & Thompson, 2000a; Steffe, von Glasersfeld, Richards, & Cobb, 1983).

Epistemic Subjects

stabi …t proves
incr … use in
pre … viors.

Steffe … s] are
usefu … an be
used t… matical
actic … 319).



- Thompson, 2000, p. 426

Epistemic Subjects

Levels

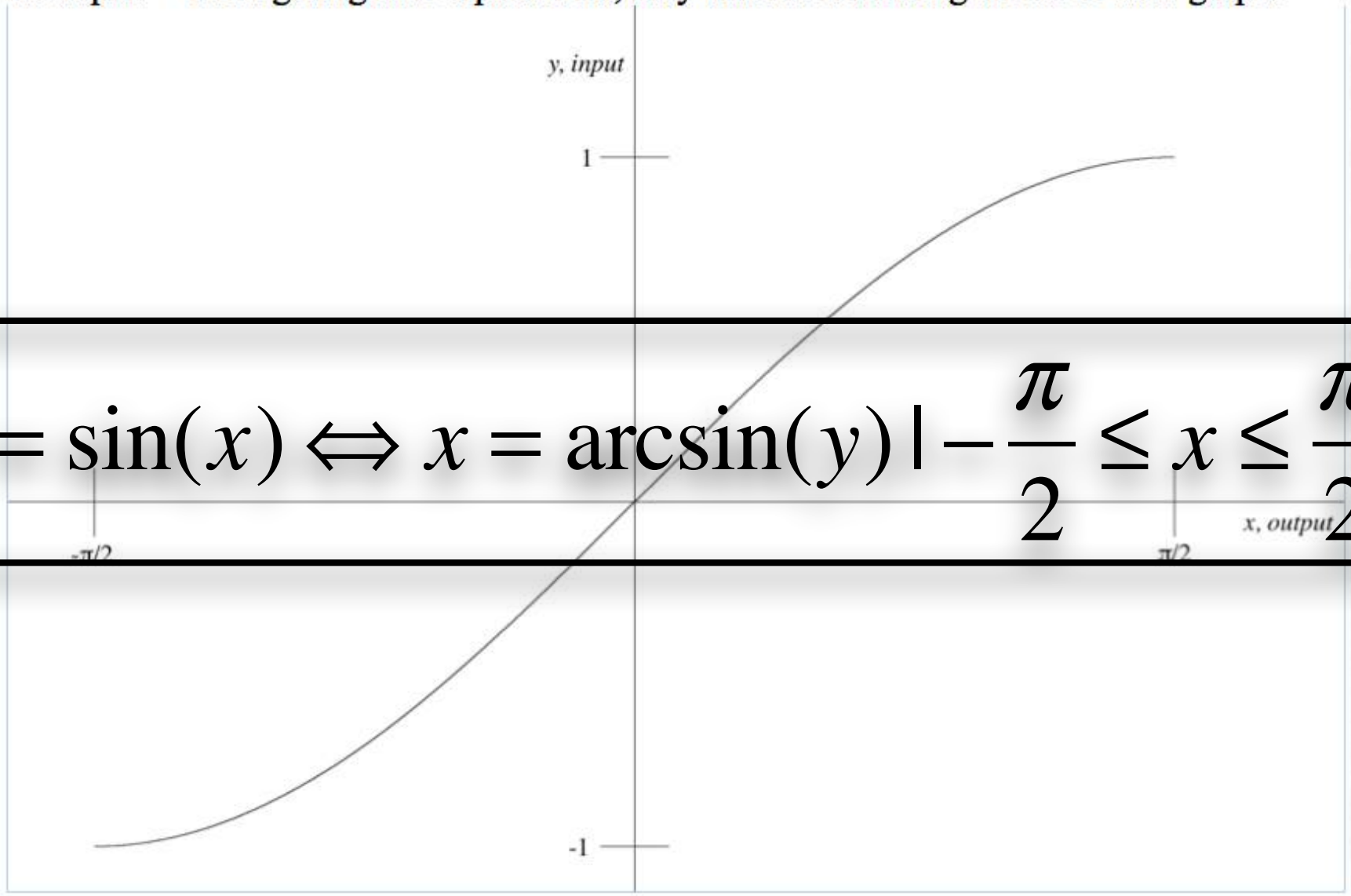
vs.

Stages

Illustrations

To what extent are the students' graphing activities dominated by figurative or operative thought?

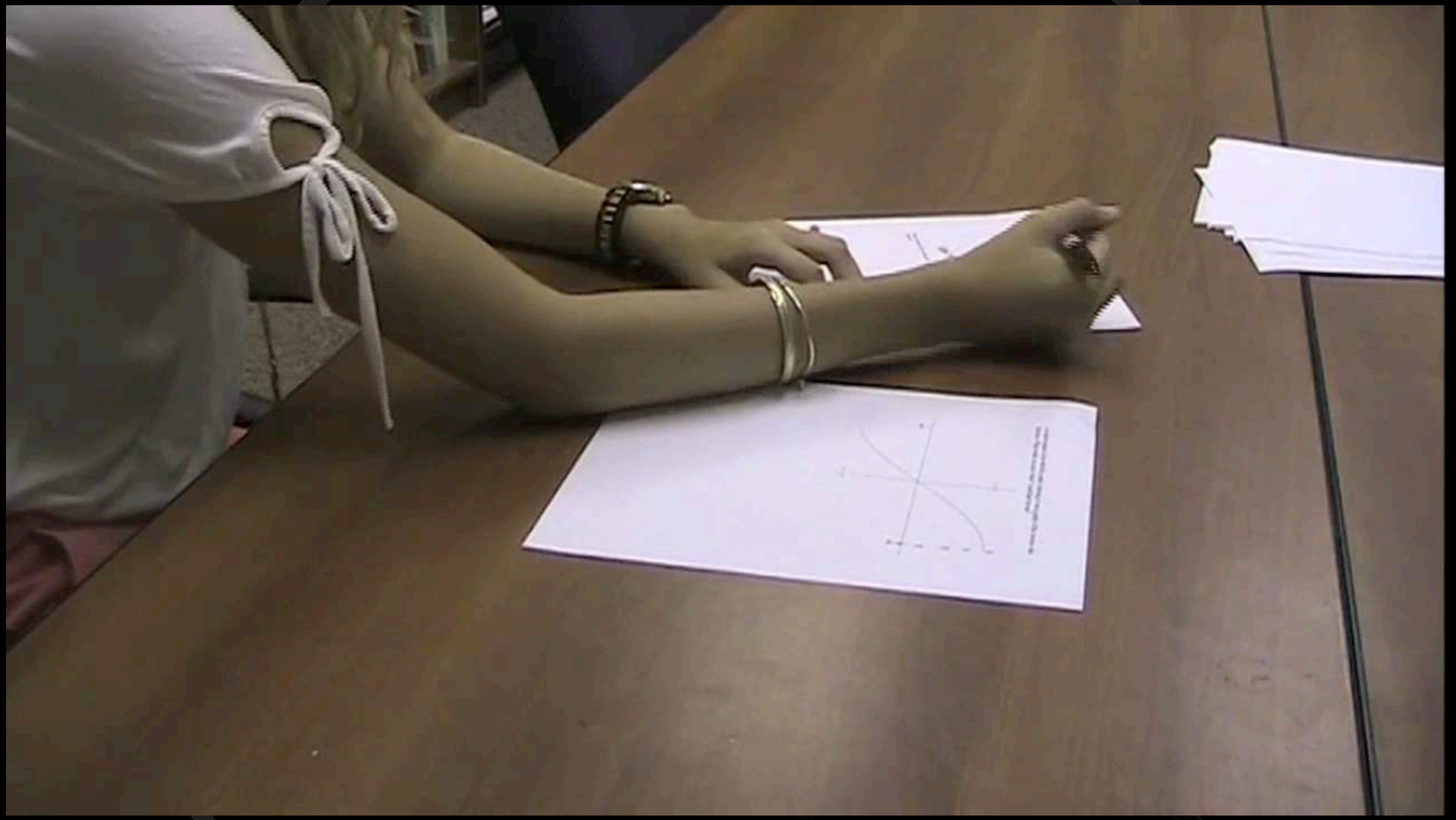
When questioning the students from the previous problem, they claim, "Well, because we are graphing the inverse of the sine function, we just think about x as the output and y as the input." When giving this explanation, they add the following labels to their graph.



$$y = \sin(x) \Leftrightarrow x = \arcsin(y) \mid -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

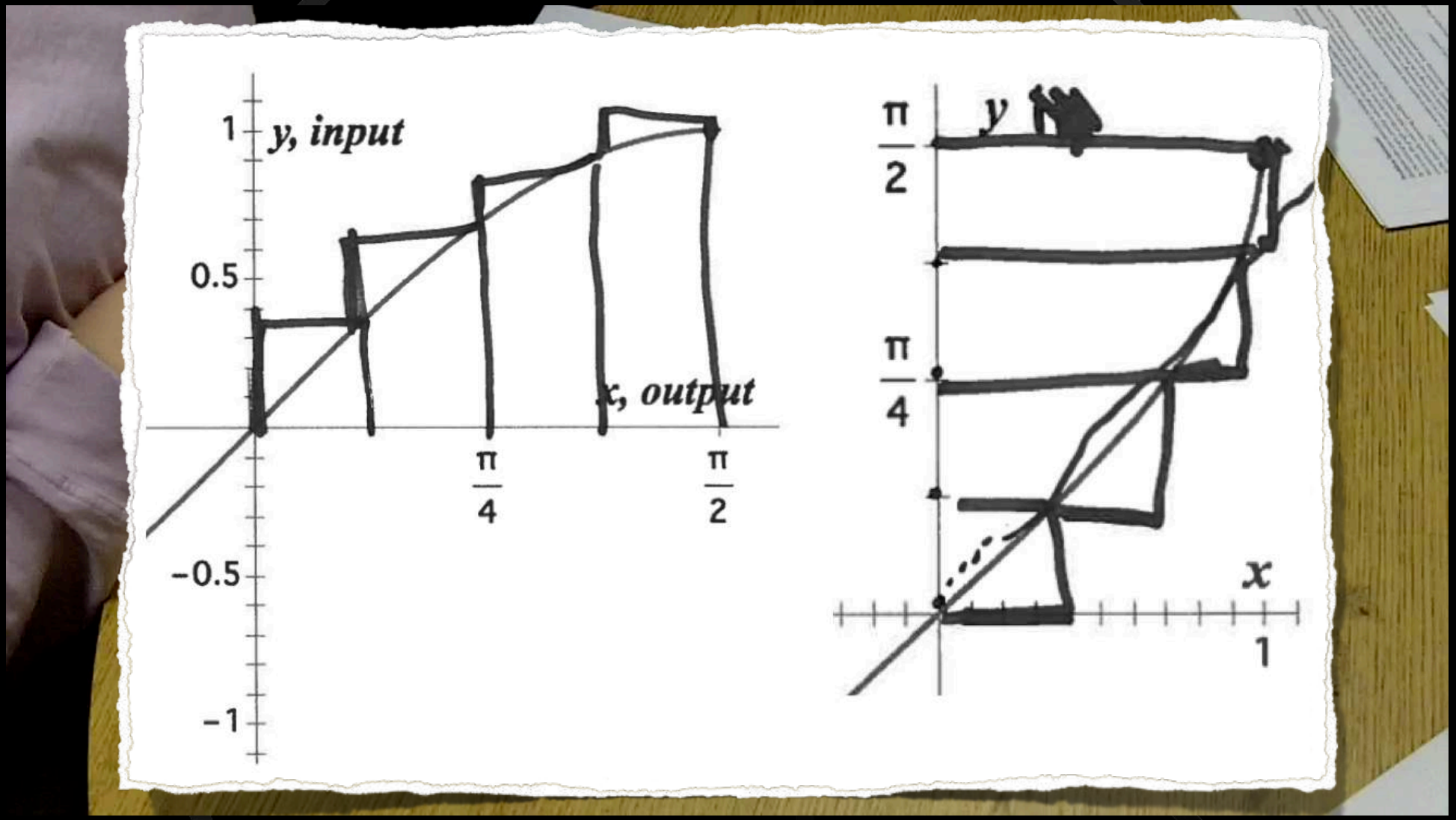
What do you think about the students' graph? Is it correct? Why or why not? How would you respond to the students?

…looks like…plain sine graph…going to be different.

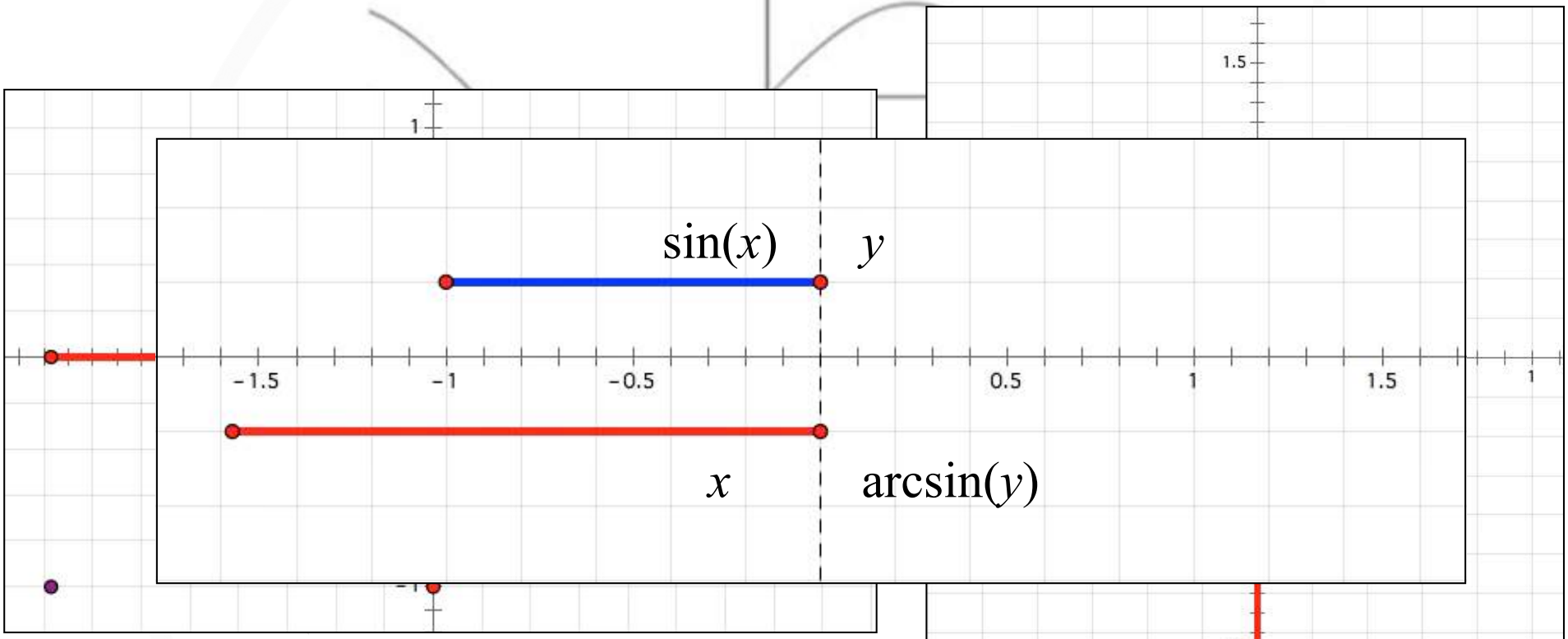


…it's a graph everyone knows about.

…increasing at a decreasing rate…

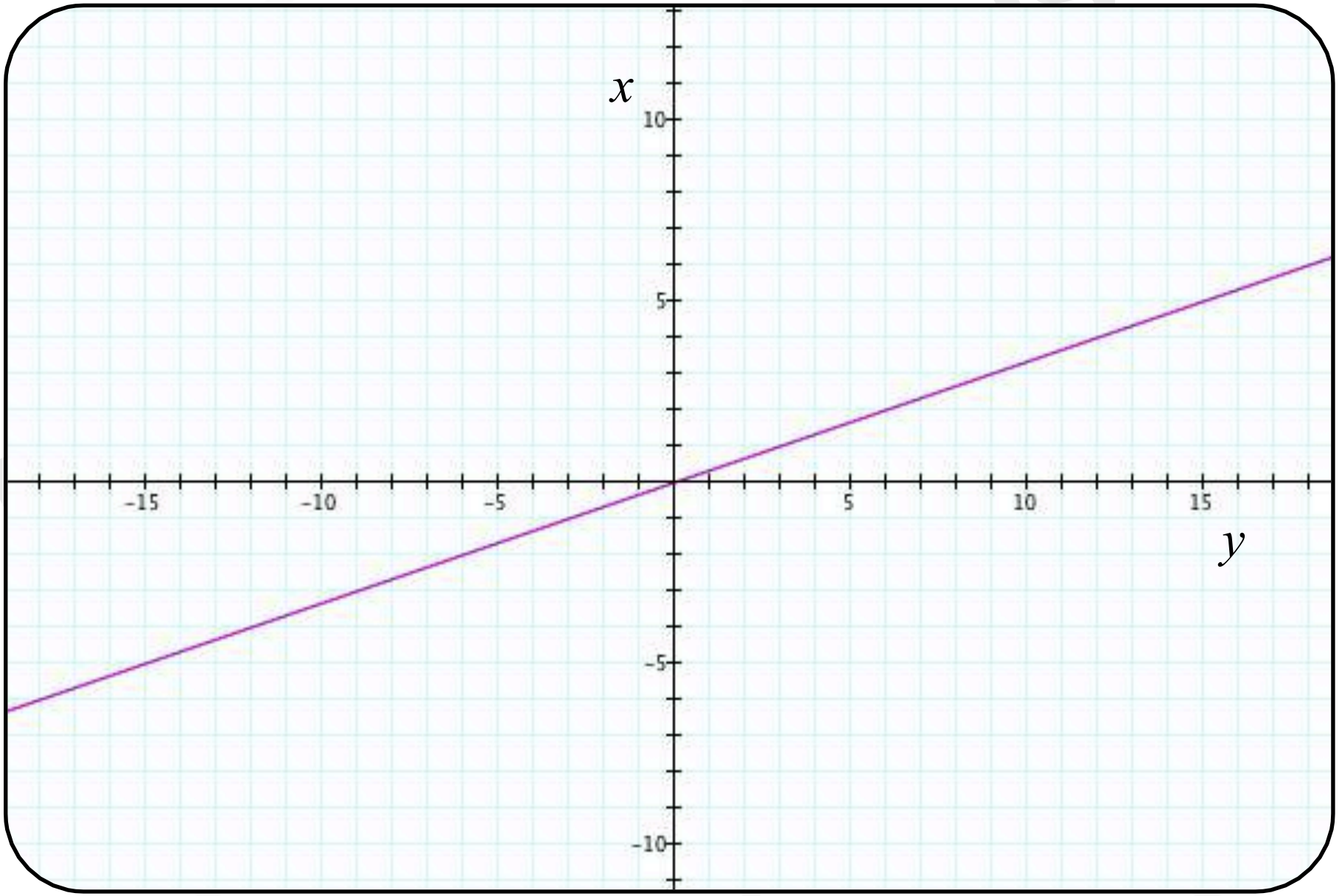


…it's still showing the same thing.

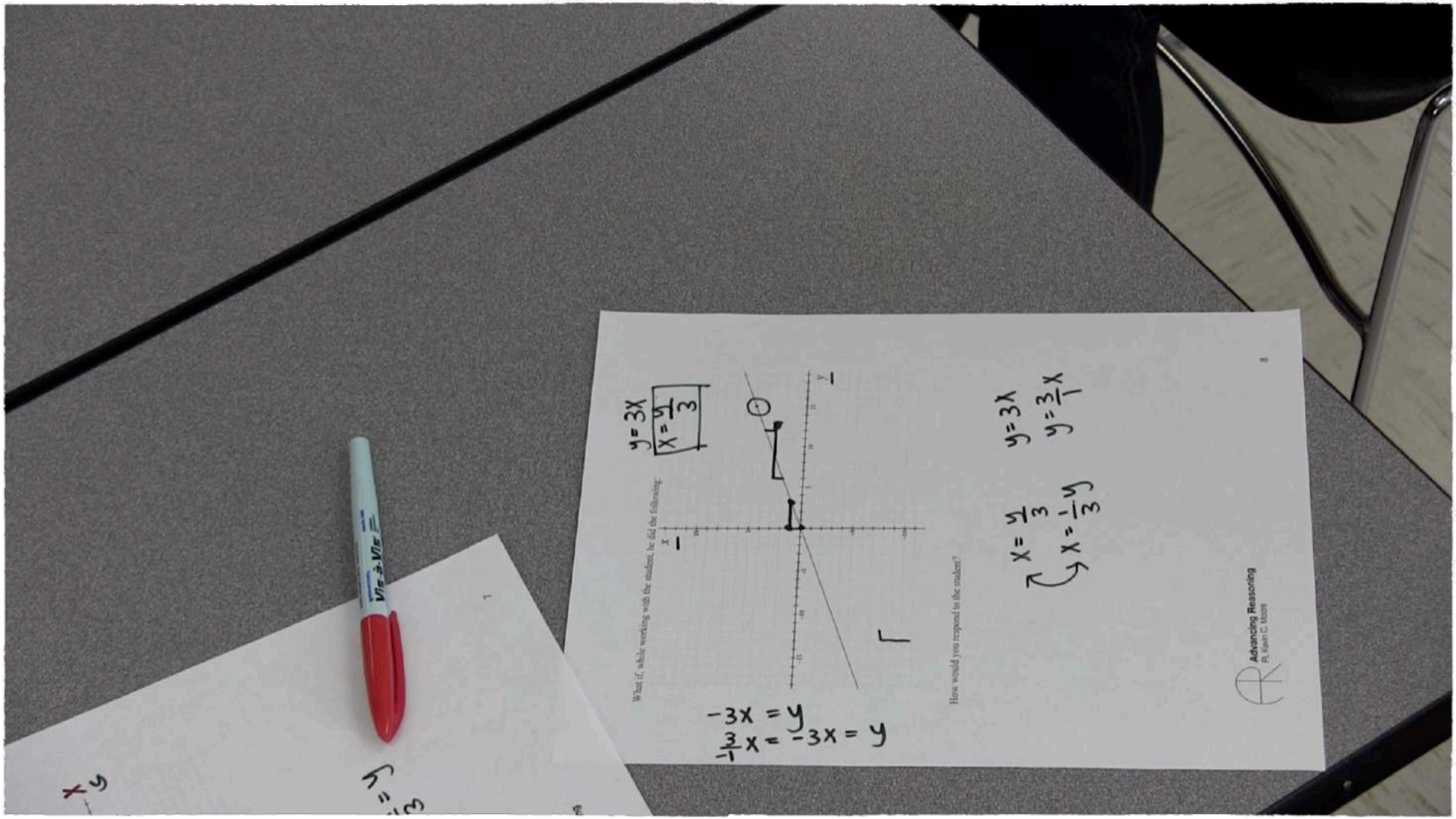


$$y = \sin(x)$$

A hypothetical student is graphing the relationship $y = 3x$…

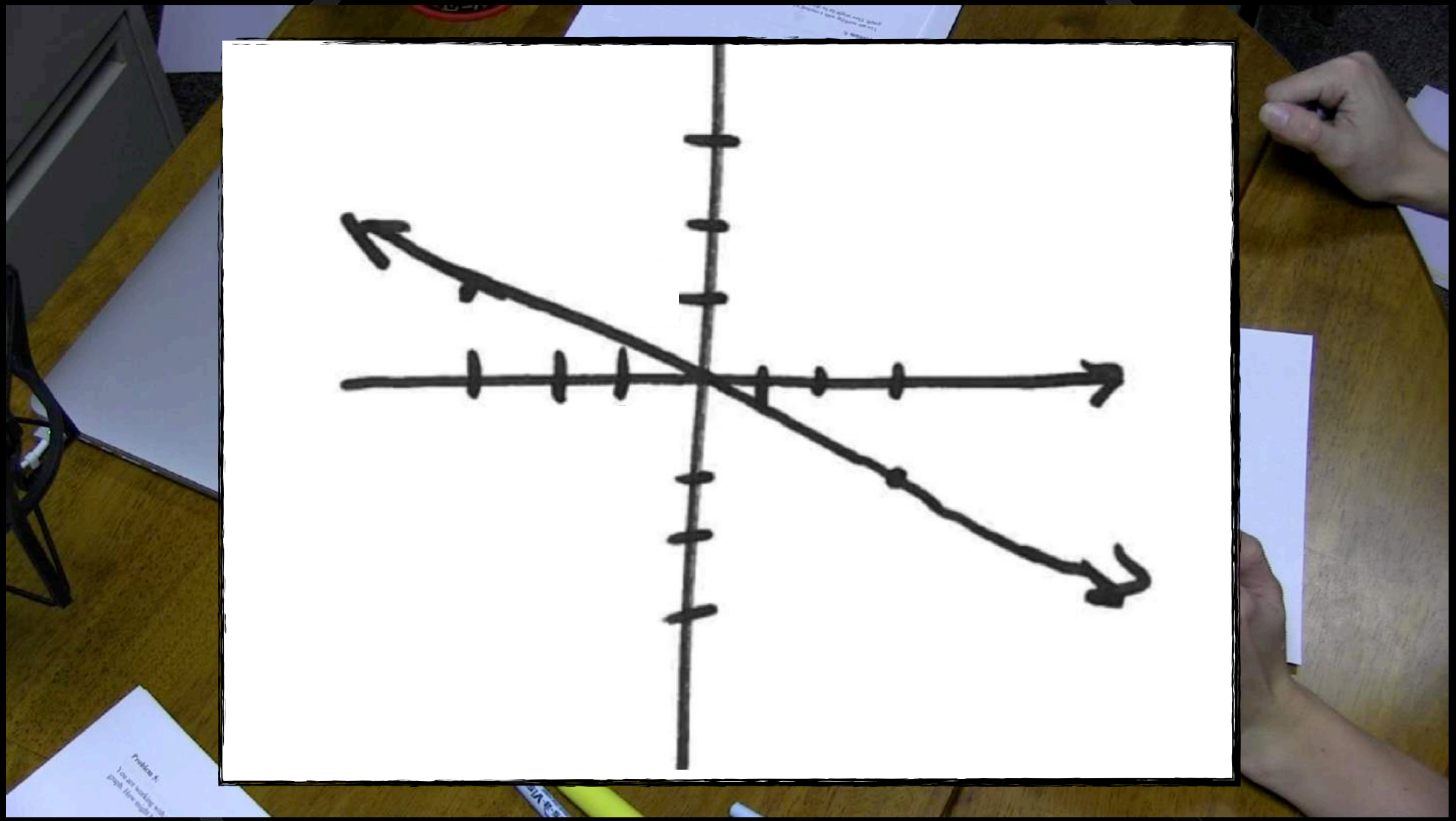


The slope is negative again…

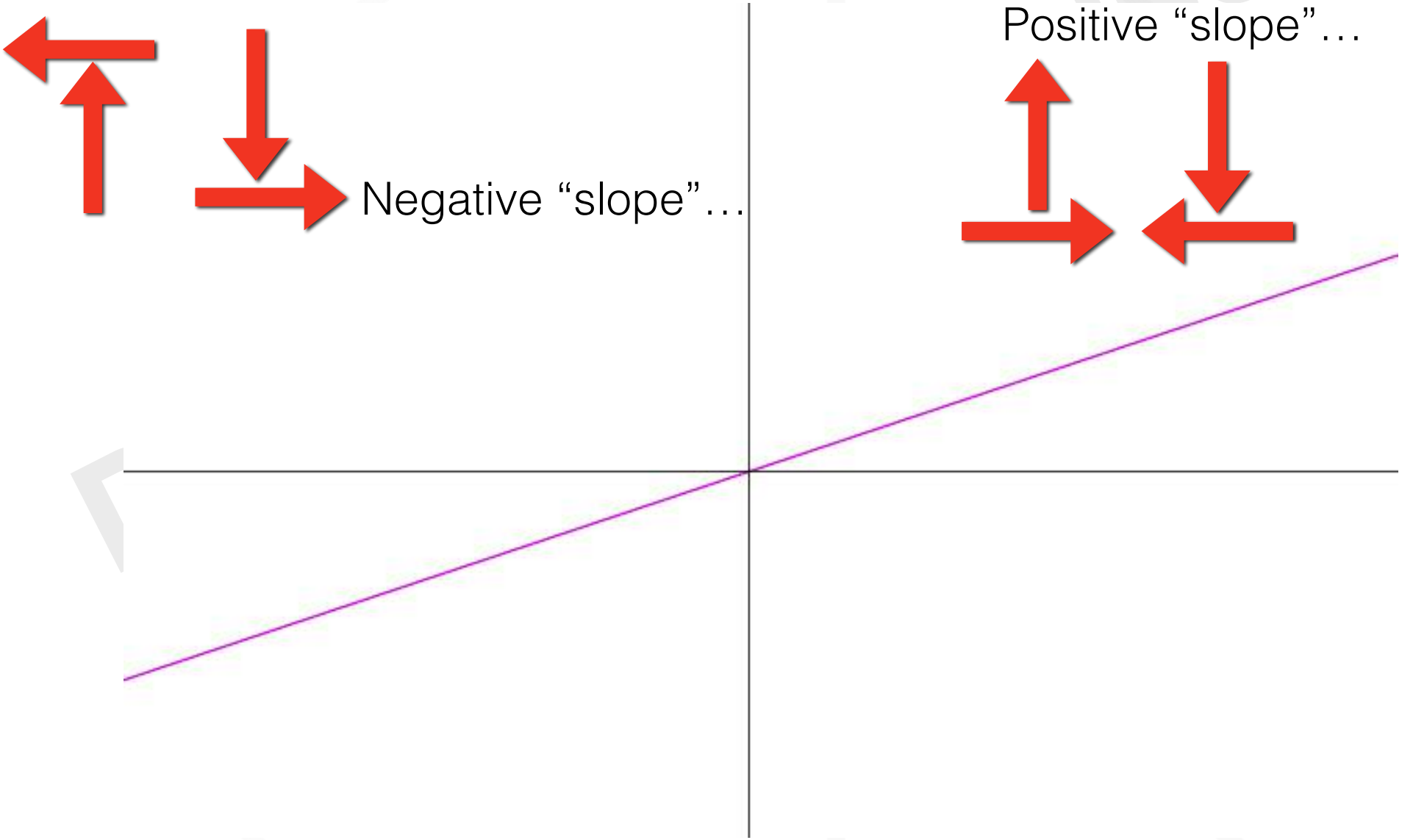


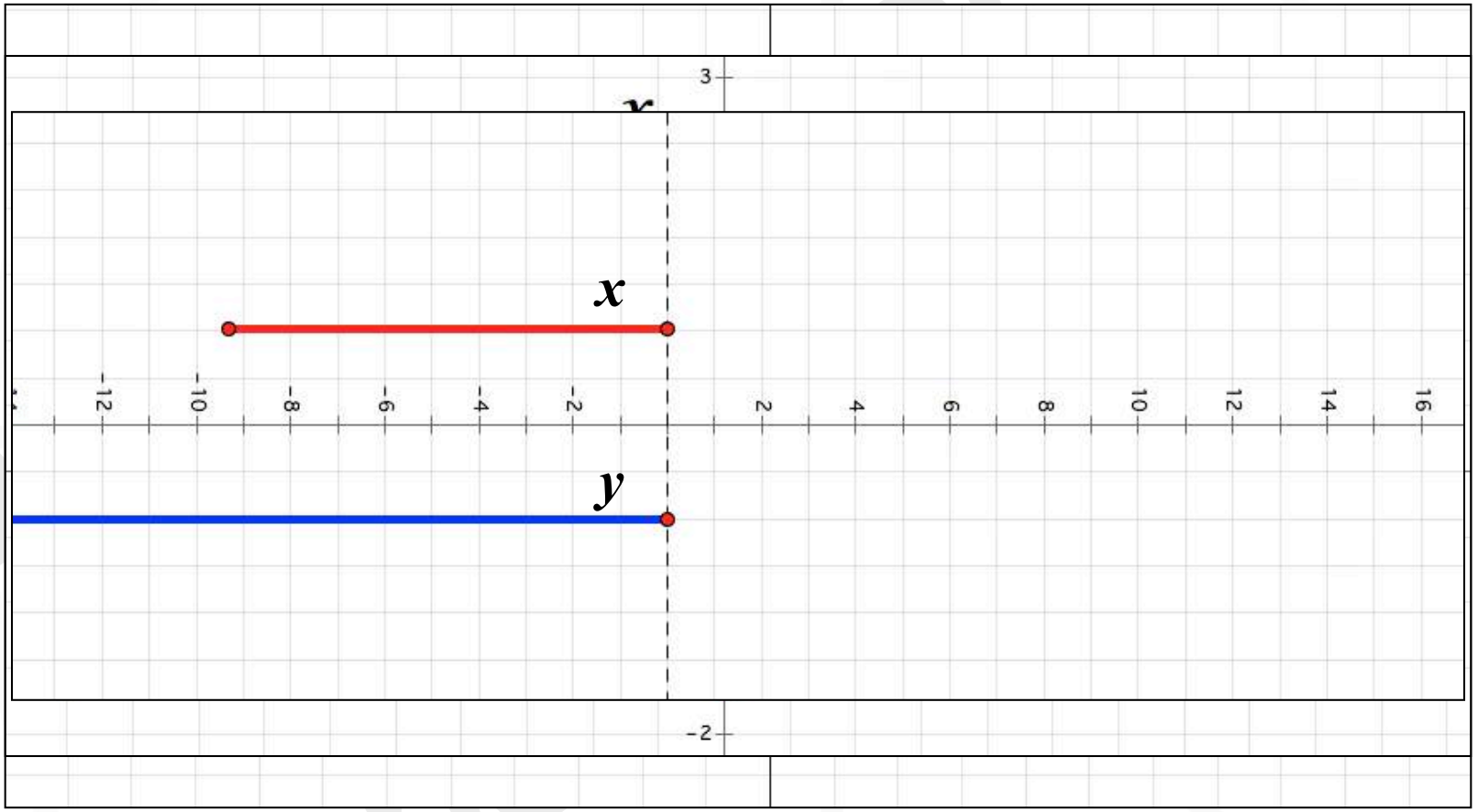
…going the negative direction.

But that's only visual…rate of change…



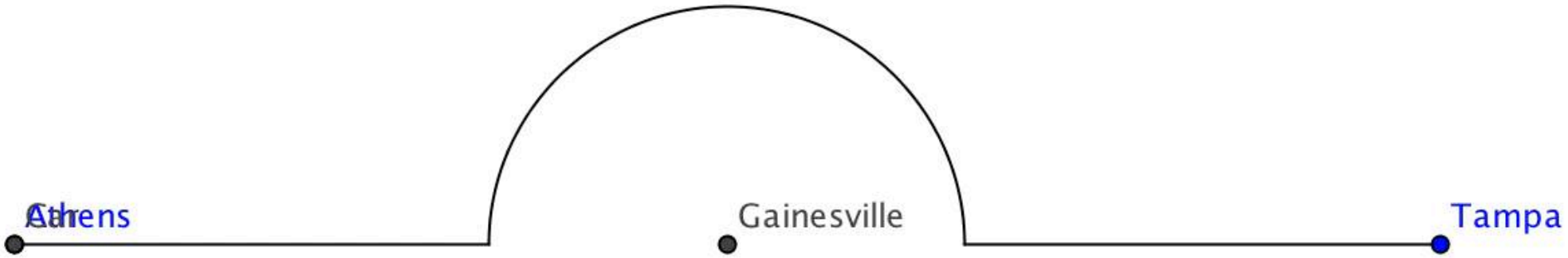
…positive increase of one…positive increase of three.


Negative “slope”…
Positive “slope”…

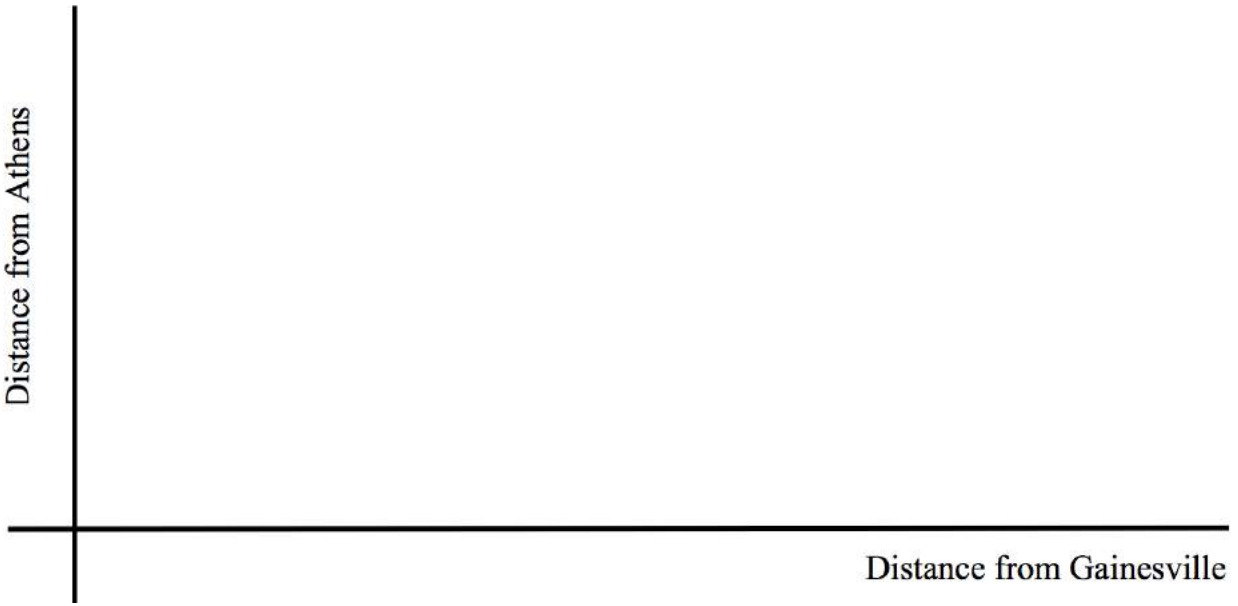


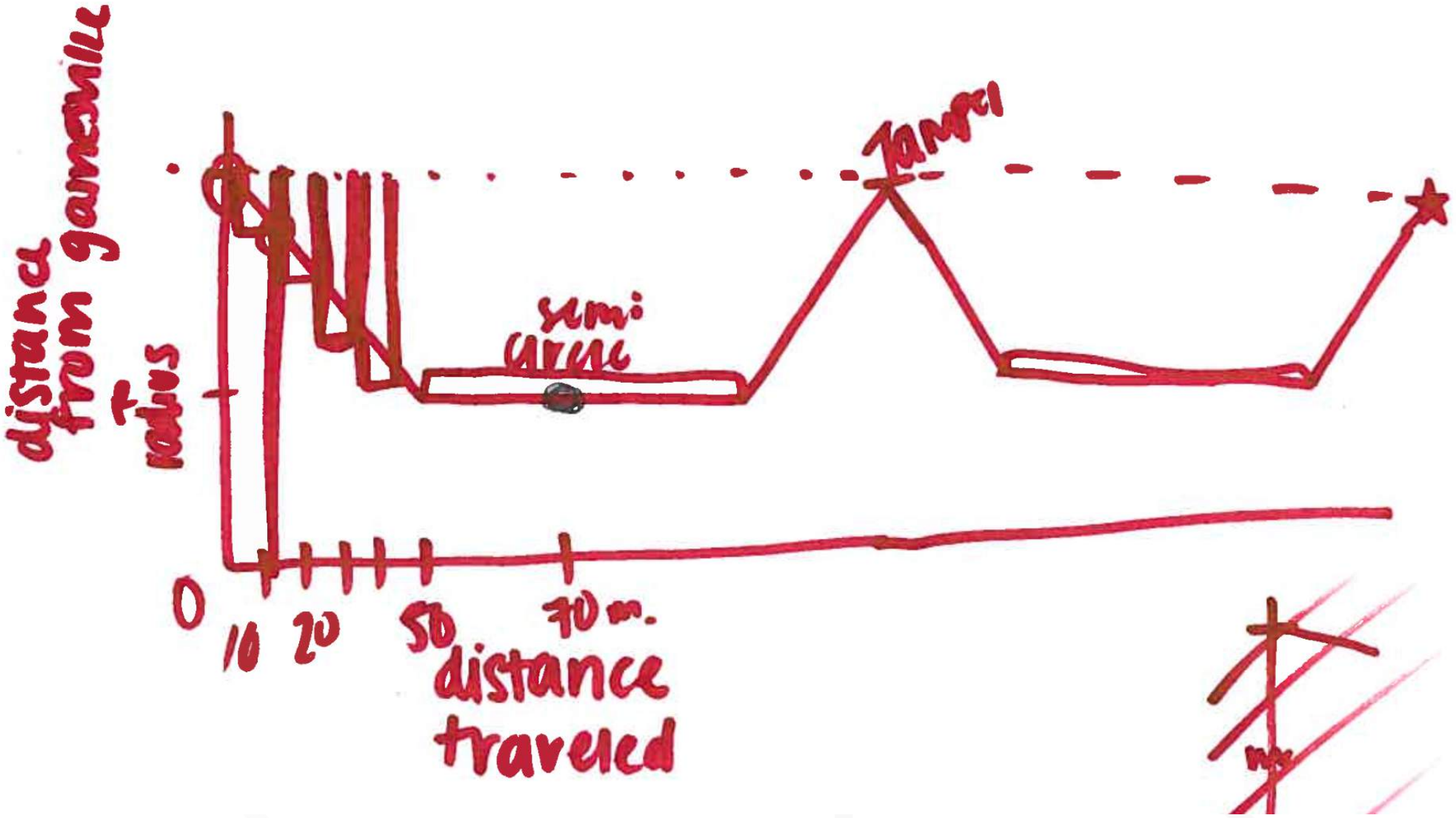
"It's backwards..."

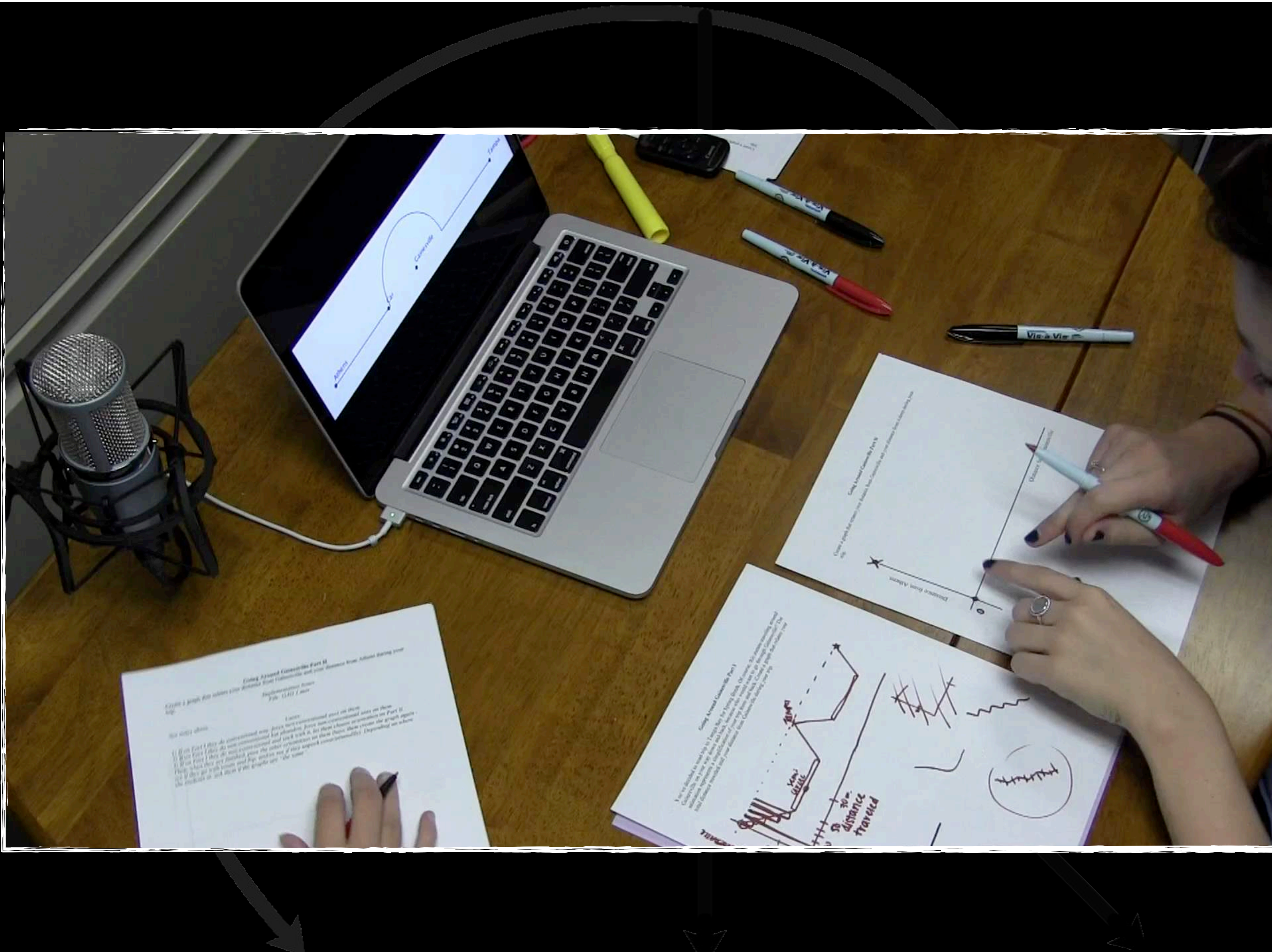
“It’s backwards.”

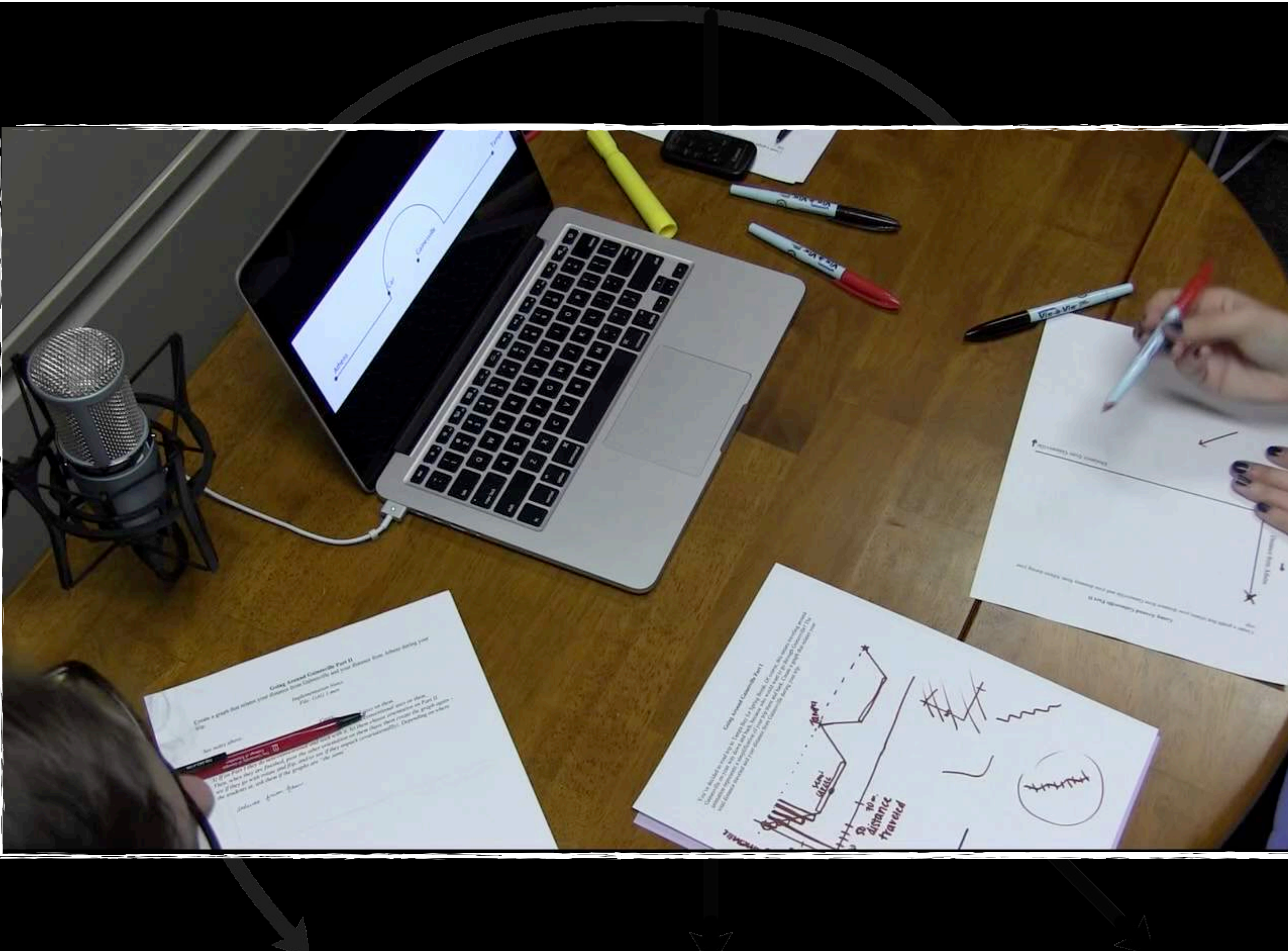


Create a graph that relates your distance from Gainesville and your distance from Athens during your trip.



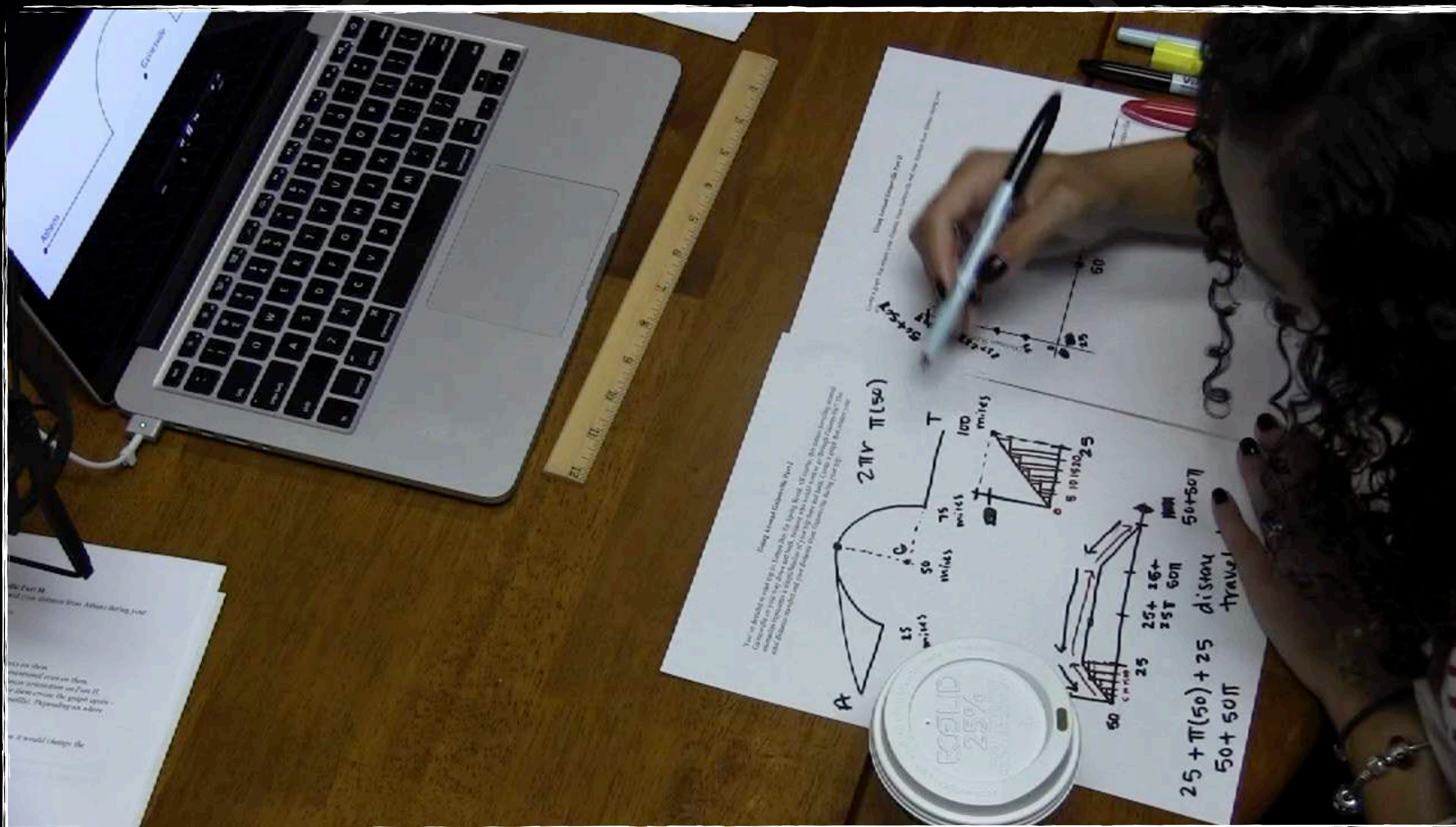

distance from gainesville
radius
Tampa
semi circle
0
10
20
50
70 m.
distance traveled

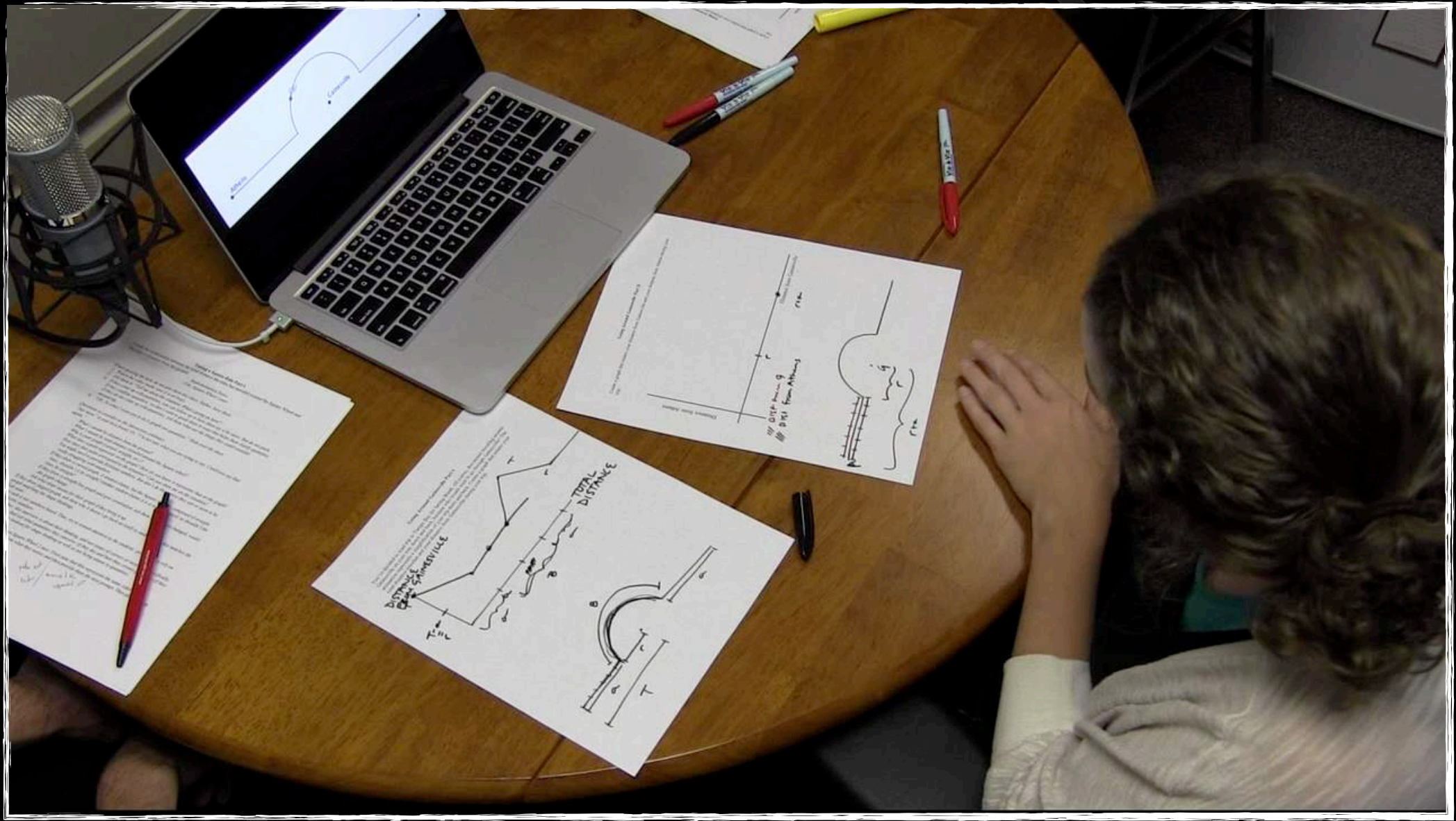

Athens
Gainesville
Tampa
distance traveled



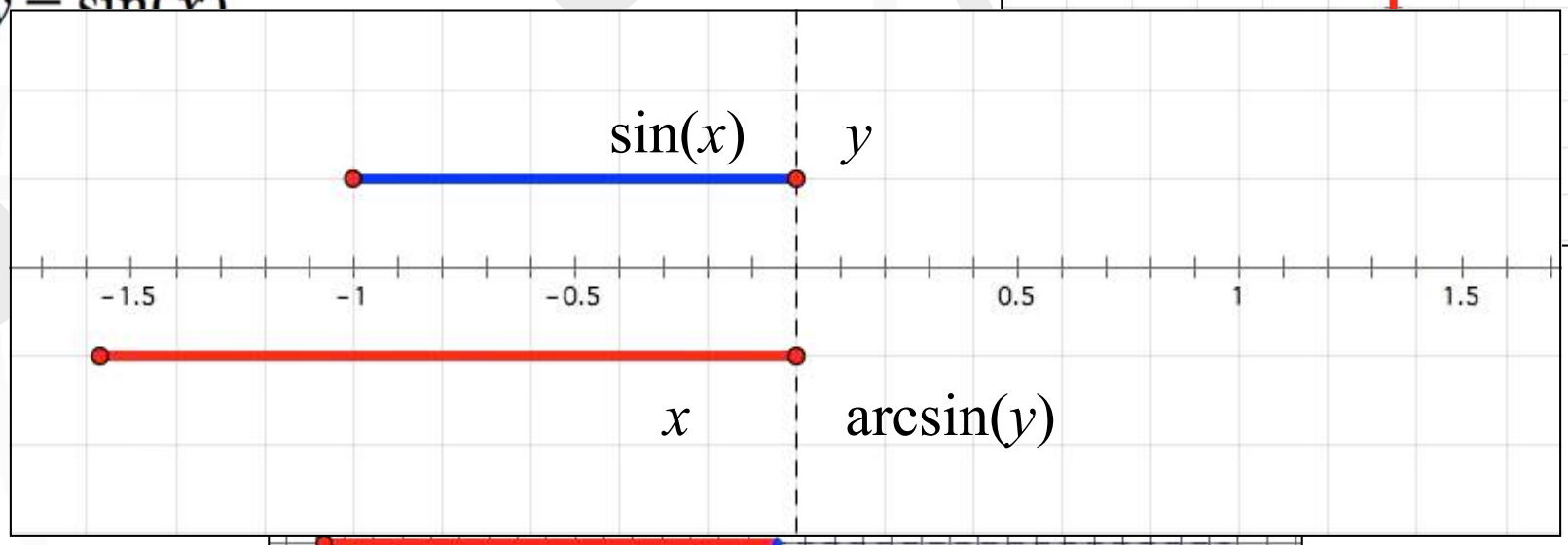
Both actions entailed ‘starting’ on the vertical axis and anticipating a graph traced left-to-right.

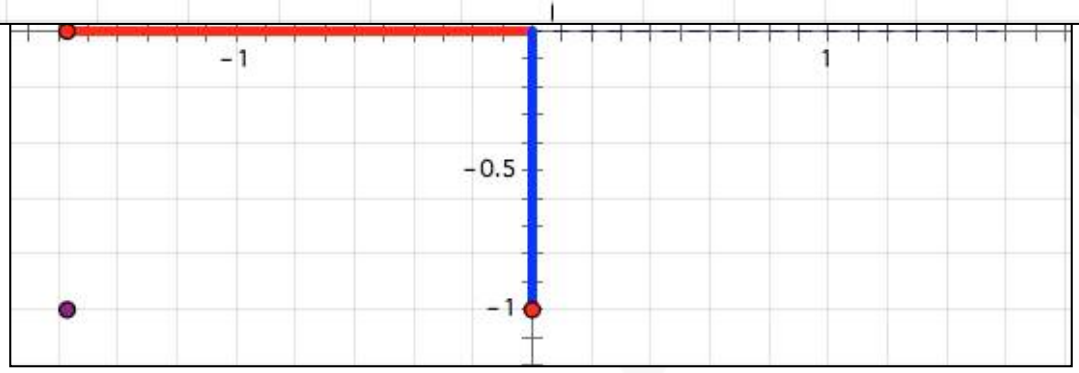
Belle held in mind a relationship she wanted to represent.




TOTAL DISTANCE

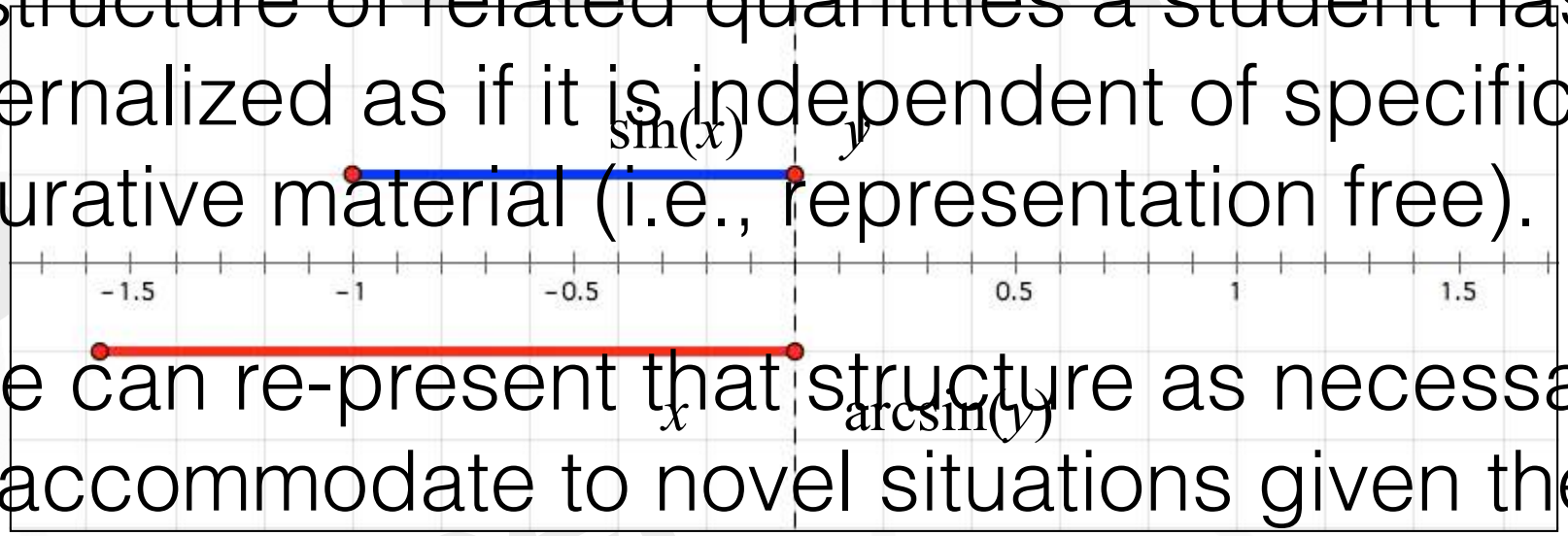
Abstracted Quantitative Structure



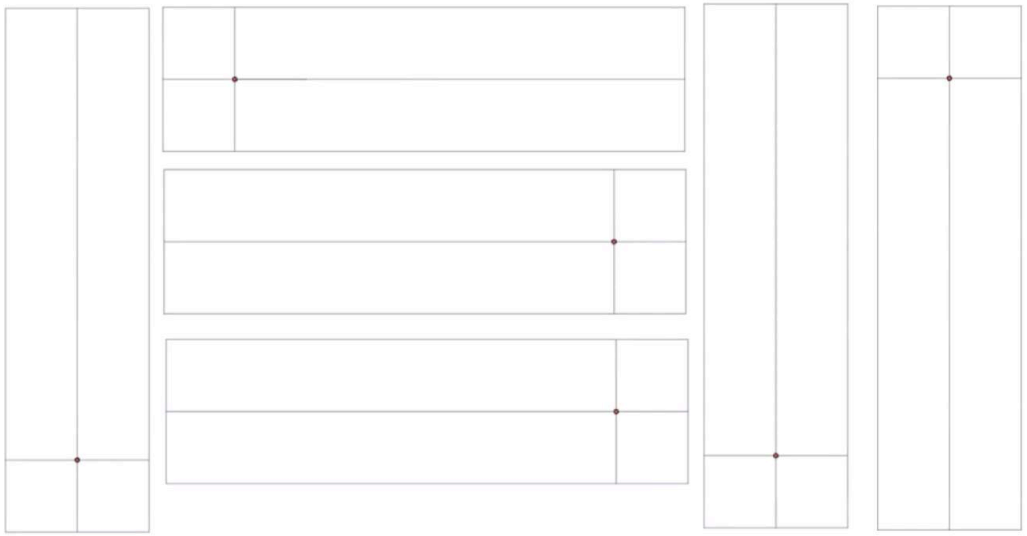


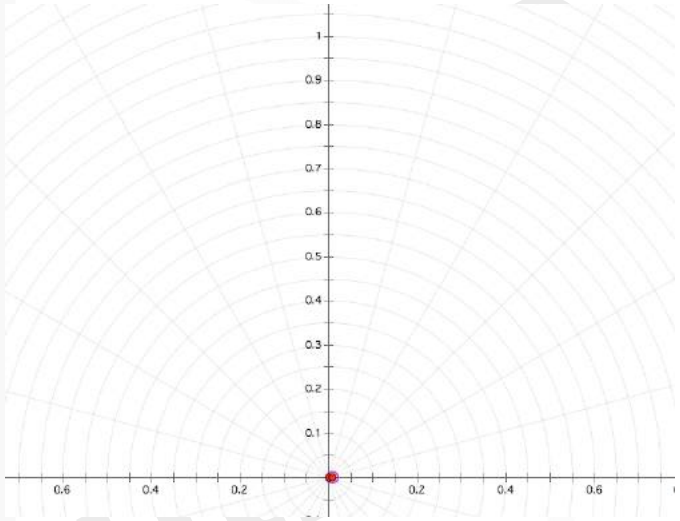
Abstracted Quantitative Structure

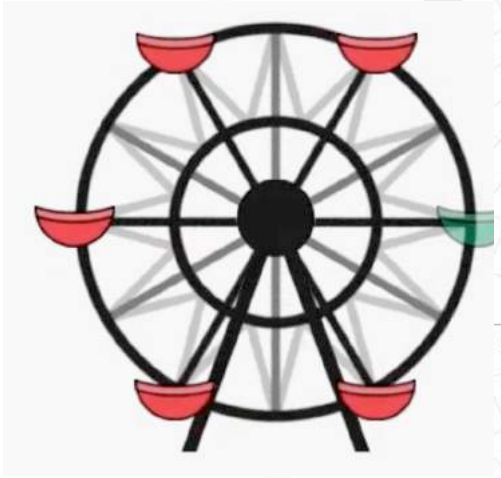
a structure of related quantities a student has internalized as if it is independent of specific figurative material (i.e., representation free).

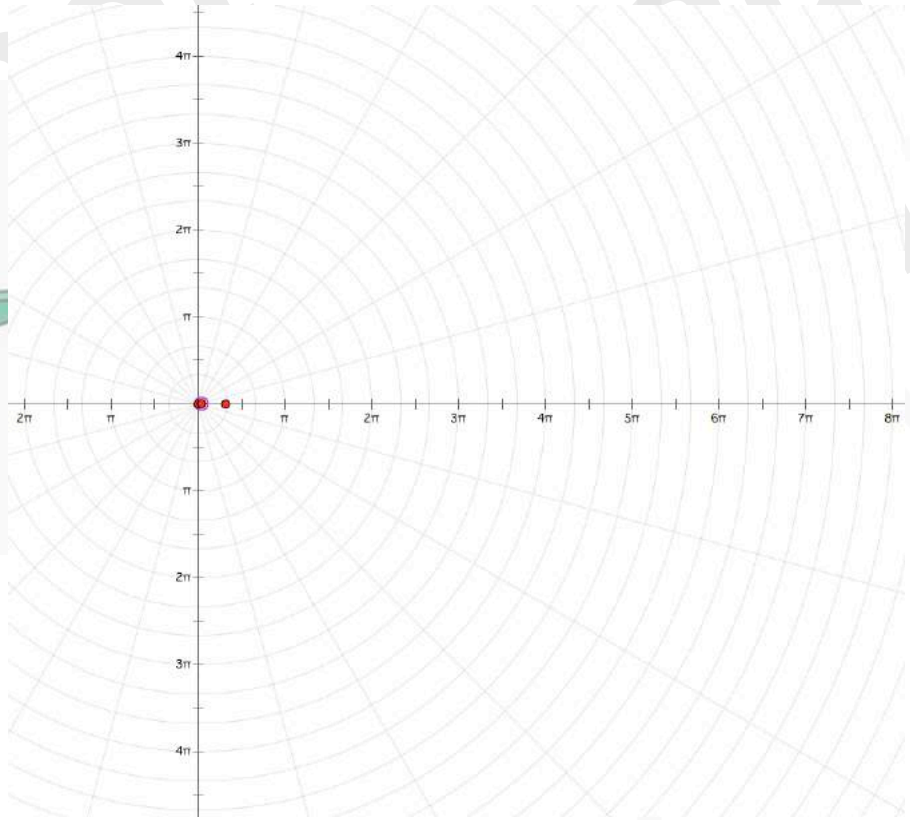


She can re-present that structure as necessary to accommodate to novel situations given the situation enables representing that structure veridically.




1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.6
0.4
0.2
0.2
0.4
0.6




4π
3π
2π
π
2π
π
π
2π
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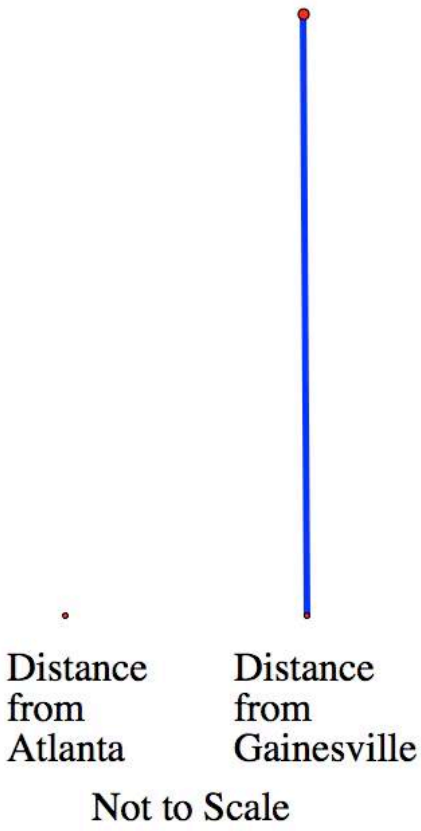
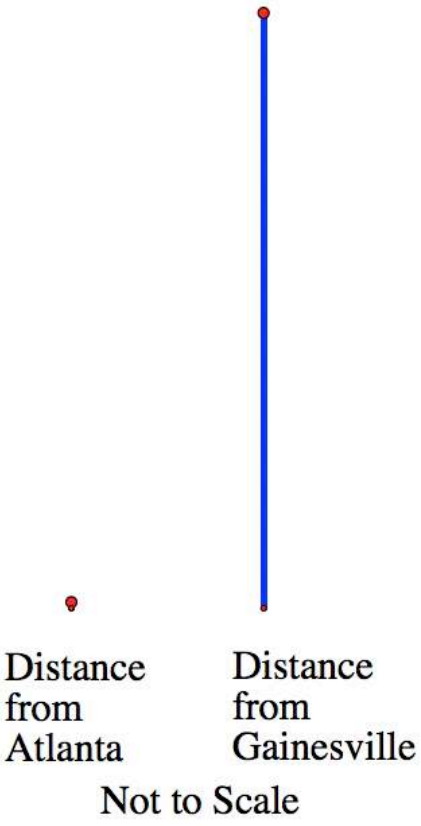
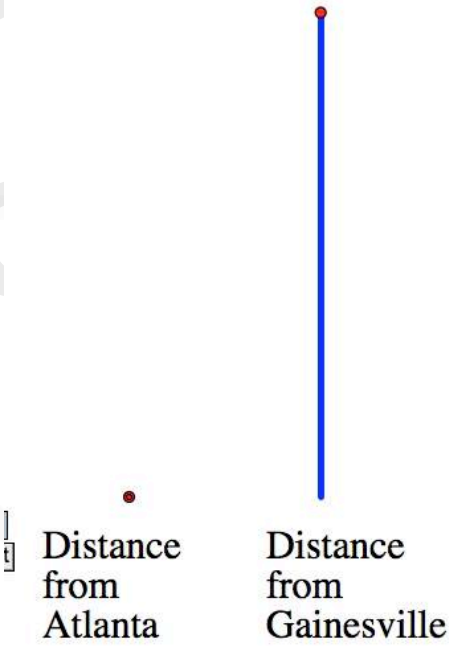
4π
5π
6π
7π
8π
π
2π
3π
4π

What Next?

Constructing and **sustaining** covariational relationships…







…and developing operative representational systems.

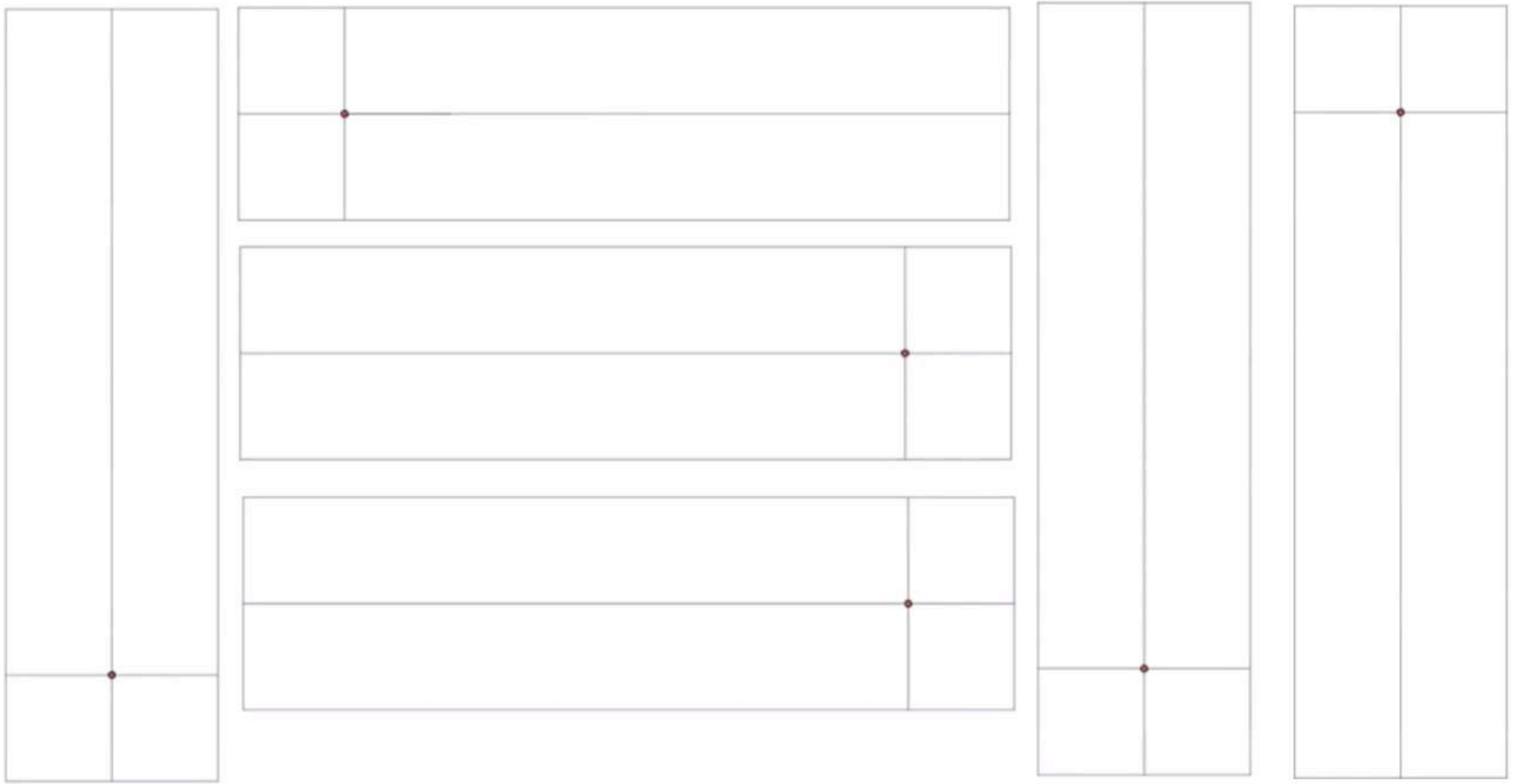
Drive
Reset
G_Horizontal
G_Vertical
A_Horizontal
A_Vertical
Join
Hide Link

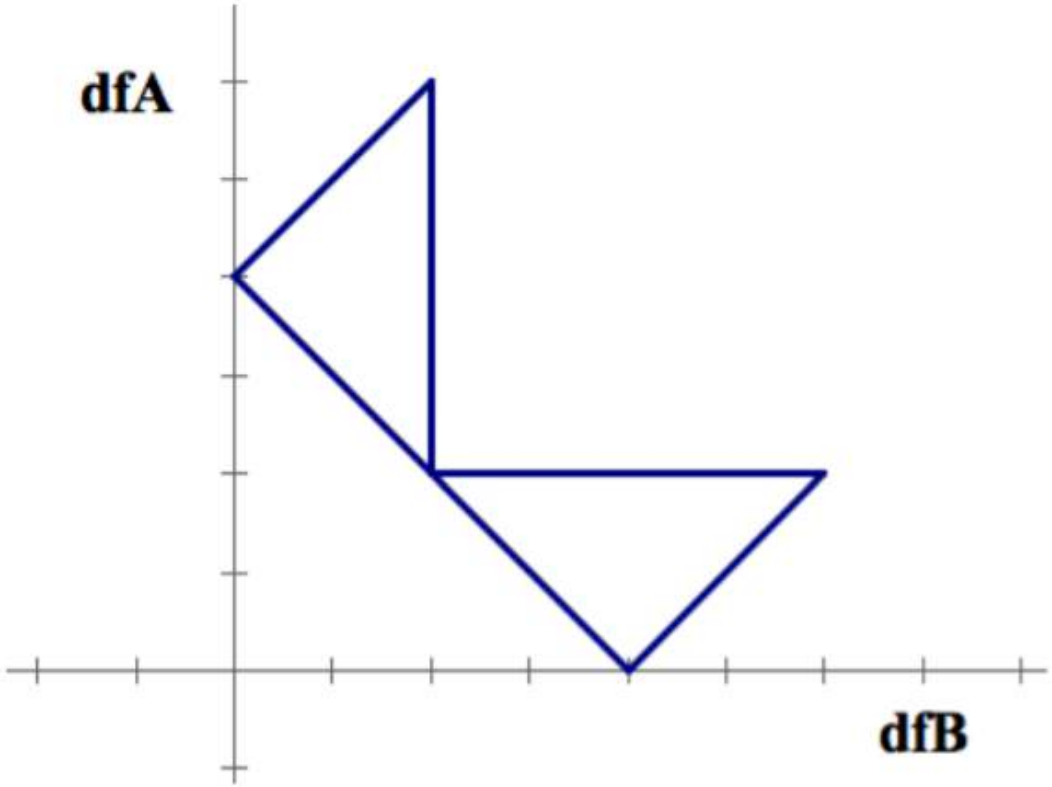
Distance from Atlanta

Distance from Gainesville

Not to Scale

Part I-G | Part II-G | Part III-G | Part I-T | Part II-T | Part III-T | Beta 1 | Beta 2 | Beta 3




dfA
dfB

THANK YOU

https://sites.google.com/site/advancingreasoning/

Biyao **Irma** **Halil** **Hamilton** **Natalie**





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