

# ASSESSING SECONDARY TEACHERS' ALGEBRAIC HABITS OF MIND

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## TODAY'S AGENDA

1. Background on our work
2. Paper and pencil assessment — Part I
  - (a) Review the items in small groups
  - (b) Whole group discussion
3. (If time allows) Paper and pencil assessment — Part II
4. Next steps
5. Further discussion and questions

## WHAT IS ASTAHM?

ASTAHM is an NSF DRK-12 collaborative project funded in 2012 aimed at developing instruments to assess secondary teachers' Mathematical Habits of Mind (MHoM).

## WHAT DO WE MEAN BY MATHEMATICAL HABITS OF MIND?

We define **mathematical habits of mind** (MHoM) to be:

*the specialized ways of approaching mathematical problems and thinking about mathematical concepts that resemble the ways employed by mathematicians.*

## THE ASTAHM TEAM: BOSTON UNIVERSITY, ST. OLAF COLLEGE, EDUCATION DEVELOPMENT CENTER

- Glenn Stevens
- Ryota Matsuura
- Sarah Sword
- Al Cuoco
- Mary Beth Piecham
- Miriam Gates
- Kristen Luce
- Russell Faux (DSRA)

ASTHAM grew out of the continuing NSF-funded MSP project,  
Focus on Mathematics.

## FOCUS ON MATHEMATICS

- Focus on Mathematics (FoM) is a Targeted Math and Science Partnership funded by the NSF since 2003.
- FoM is a unique partnership of teachers, students, administrators, mathematicians, and mathematics educators—all committed to increasing student achievement.
- FoM is a collaboration between BU, EDC, and an array of Massachusetts universities and public school districts.

## FOM GOALS

- Provide teachers with:
  - coherent, content-focused professional development,
  - sustained immersion in mathematics,
- Develop mathematically expert teacher leaders who share their knowledge with teachers and students,
- Build a mathematical learning community in which teachers and mathematicians work together, doing mathematics, and
- Improve student achievement.

## THE FOM PARTNERSHIP HAS CREATED

- School based study groups
- Seminars, colloquia, and summer institutes
- New graduate degrees
- Online problem solving courses for teachers
- Avenues for teacher leadership
- Student mathematics fairs (over 10,000 students participating)
- Research collaboratives
- Case studies of participating teachers
- **Research study to measure MHoM as part of Phase II**



## AUTHENTIC MATHEMATICAL EXPERIENCES

### **Experience first:**

It has been observed in every human activity experience comes first, and as this experience grows the need for communication motivates the development of language. Sadly enough, in our classroom practice we place language first and experience second. We worry about what we should say in order to help the student “understand.” By this we mean to provide the effect of experience through the use of suitably chosen words. Not unexpectedly, the effect is at best a very pale image of the real thing.

— Arnold Ross

## IMPACT ON TEACHERS

FoM teachers have reported<sup>1</sup>:

- Deeper knowledge of mathematics
- Changes in beliefs about the nature of mathematics and how students learn mathematics
- Renewed passion for mathematics
- Changes in instructional practice, for example
  - use of precise language
  - connecting mathematical ideas
  - mining student ideas and approaches

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<sup>1</sup>Baldassari, C., Lee, S., & Torres, R. T. (2009). *The case of a high school mathematics teacher*. Retrieved from <http://focusonmath.org/FOM/PERG>

## INITIAL MOTIVATION FOR RESEARCH

- Through our FoM work, we've seen that MHoM is indeed a collection of **habits teachers can acquire**, rather than some static you-have-it-or-you-don't way of thinking.
- And teachers reported to us that developing these habits has had a **tremendous effect on their teaching**.
- We recognized **the need for scientific-based evidence** to establish that these teachers have indeed learned MHoM and that these habits have had a positive impact on their teaching practice.
- **The instruments to measure these habits did not yet exist.**

## MET2 FRAMEWORK FOR MKT

The MET2 framework uses four large and overlapping categories to characterize some of the ways in which teachers know and understand mathematics:

- (1) As a scholar
- (2) As an educator
- (3) **As a mathematician**
- (4) As a teacher

## KNOWING MATHEMATICS AS A MATHEMATICIAN

From our experience, we believe that (3) knowing mathematics as a mathematician. . .

- enriches and enhances the other ways of knowing mathematics,
- can bring efficiency and coherence to teachers' mathematical thinking and to their work with students,
- and thus is an important aspect of **mathematical knowledge for teaching at the secondary level.**

## WHAT WE'RE REALLY STUDYING

So, what we're **really** studying is the intersection of:

- (3) knowing mathematics as a mathematician,
- (4) knowing mathematics as a teacher.

## RESEARCH QUESTION

In FoM, Phase II, we began an exploratory study centered on the following research question:

*What are the mathematical habits of mind that secondary teachers use, how do they use them, and how can we measure them?*

## INSTRUMENTS FOR CONDUCTING RESEARCH

To investigate our research question, we've been developing:

- Detailed definition of MHoM, based on existing literature, our own experiences as mathematicians, and classroom observations.
- A paper and pencil (P&P) assessment that measures how teachers engage MHoM when doing mathematics for themselves.
- An observation protocol measuring the nature and degree of teachers' use of MHoM in their classroom work.

**Important remark:** We've seen the need for both instruments, and also the value of developing all three components together.



## WHAT WE AREN'T STUDYING

There are many aspects of teaching that we value but we are *not* studying right now. For example:

- Teachers' dispositions (at least not directly)
- Teachers' beliefs
- Classroom discourse

## WHAT WE AREN'T CREATING

We are *not* creating an assessment that we anticipate can say much about an individual teacher. **Our goal is to create tools for research.**

## ASSESSMENT DEVELOPMENT

Since 2010:

- Several rounds of design, small scale pilot tests (w/ FoM community), data analysis, and revision of the instrument.
- Inter-rater reliability testing for some items ( $\kappa$  up to 0.906) and concurrent rubrics refinement.
- Problem-solving think-aloud with teachers, educators, and mathematicians. (Do their interpretations match our intended objectives for each item?)

## PILOT TEST IN 2011

We conducted an “external” pilot test in 2011.

- 43 teachers ninth grade Algebra 1 teachers
- Studying implementation of a new NSF (MHoM) curriculum, *CME Project*
- 7 items (topics in secondary mathematics – algebra and geometry), to be completed in one hour.
- Cronbach alpha testing data suggests the instrument is reliable about 68% of the time.

## ASTAHM PILOT TESTS 2012

We conducted four additional pilot tests in 2012.

- ~70 teachers and ~15 “others”
- Geographic diversity
- Two part assessment:
  - 10 short items to be completed in one hour
  - 1 longer item to be completed in 30 minutes

## RUBRIC DEVELOPMENT

Initial work on creating rubrics for the P&P assessment:

- “Constant comparative analysis”
- Creating preliminary coding schemes and revising with analysis
- Seeking “recurring regularities”

(Erickson; Glasser & Strauss; Guba; Maxwell)

**Also: Barbara Scott Nelson, Heather Hill**

## MHoM: STRUCTURE AND LANGUAGE

Our current focus is on two categories of MHoM:

- Seeking, Using, and Describing Mathematical Structure (SUDS)
  - SUDS1. Discovering structure that is not apparent at first.
  - SUDS2. Making use of structure to solve problems.
- Using Mathematical Language (LANG)
  - LANG1. Using language to acquire experience, clarity, and understanding.
  - LANG2. Exercising appropriate “mathematical hygiene” (e.g., using language precisely).

**Note:** Eventually, we will investigate other habits, e.g., “Performing purposeful experiments” and “Applying mathematical reasoning.”

## P&P ASSESSMENT: KEY FEATURES

Distinguishing features of the P&P assessment:

- **It measures how secondary teachers use MHoM in their own doing of mathematics, in familiar contexts.**
- Content is from secondary mathematics—i.e., mathematical problems that most teachers have the requisite knowledge to solve, or at least begin to solve.
- **We're interested in their approach, as opposed to whether or not they can arrive at a solution.**
- Our items are drawn from multiple sources, including our classroom observation work.



## FOR YOU TO DO

In small groups of ~4 people, please review the P&P items assigned to your group and discuss the guiding questions in the handout.

- Group A: Items 1, 2, 3.
- Group B: Items 4, 5, 7.
- Group C: Items 5, 6, 7.
- Group D: Items 8, 9, 10.

Afterwards, we'll ask each group to report on your conversations.

## NEXT STEPS ON THE P&P

### What's next?

- Upcoming rounds of smaller field tests (we just can't stop!)
- Validity & Reliability testing of the assessment
- Larger field test 2014-2015: 200 teachers
- Student data collection (PARCC assessments)

## THANK YOU

- If you have further feedback and/or questions, please email me:

*ssword@edc.org*